Time lags and reverberation in the lamp-post geometry of the compact corona illuminating a black-hole accretion disc

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Title: **Probing Strong Gravity by Black Holes Across the Range of Masses**

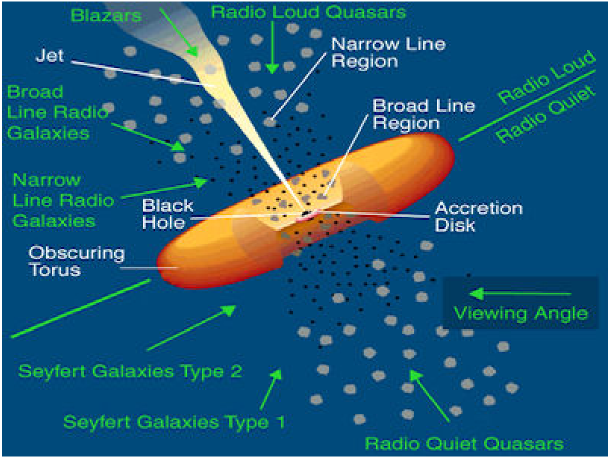
Institutes: AsU, CNRS, UNIROMA3, UCAM, CSIC, UCO, CAMK

Webpages: `http://stronggravity.eu/`

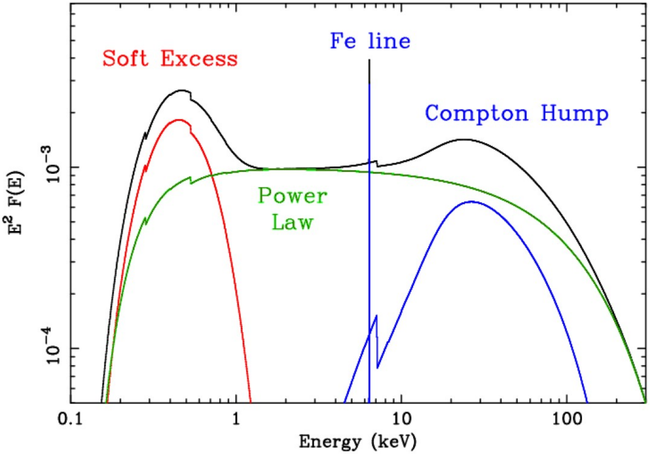
`http://cordis.europa.eu/projects/rcn/106556_en.html`

SEVENTH FRAMEWORK PROGRAMME

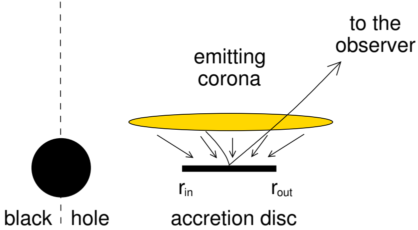
Active Galactic Nuclei – scheme

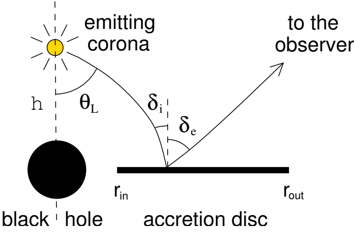


Active Galactic Nuclei – X-ray spectrum



Innermost part of AGN – scheme





Relativistic iron line

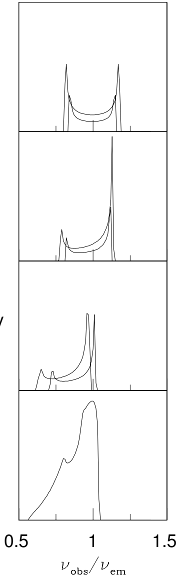
$$G(g) = \sqrt{\frac{g_+\, g_-}{(g_+ - g)(g - g_-)}}\, g^2 \cos\left[\theta_e(g)\right] \ell(\mathrm{g}) \left[\frac{\partial \psi}{\partial \varphi}(g)\right]^{-1}$$

Newtonian

Special relativity

General relativity

Line profile



Fabian et al. (2000)

Transverse Doppler shift

Beaming

Gravitational redshift

$$g = 1 - \frac{v_k}{c} \sin\theta_o \sin\varphi$$

$$\theta_e = \theta_o \quad \ell = 1 \quad \psi = \varphi$$

$$g = \left[\gamma\left(1 + \frac{v_k}{c} \sin\theta_o \sin\varphi\right)\right]^{-1}$$

$$\theta_e = \theta_o \quad \ell = 1 \quad \psi = \varphi$$

$$g = \left[U_k^t \left(1 - \Omega_k \sin\theta_o\, \alpha\right)\right]^{-1}$$

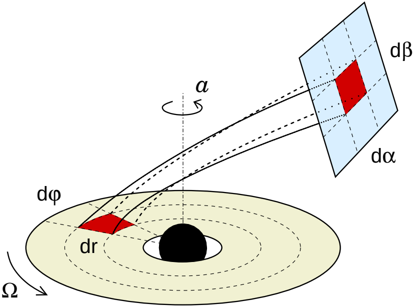
$$\cos\theta_e = \sqrt{\beta^2 + (\alpha^2 - a^2)\cos^2\theta_o}\,/r$$

$$\ell = \frac{\mathrm{d}S_o}{\mathrm{d}S_{loc}}$$

$$\sin\psi = \frac{2(\alpha - \overline{\alpha})}{\alpha_+ - \alpha_-}$$

$$F(g) = \int_{r_{in}}^{r_{out}} \mathrm{d}r\, r\, R(r) \sum_{j=1}^{2} G_j(g)\, M_j(\mu_i, \mu_e)$$

Transfer function for spectrum



$$\Delta I = \int_{\Sigma_{\alpha,\beta}} d\alpha\, d\beta\; \underbrace{g^{\gamma}}\; I_{loc}(\alpha,\beta)$$

$$\Delta I = \int_{\Sigma_{r,\varphi}} r\, dr\, d\varphi\; \underbrace{g^{\gamma-1}\mu_e\,\ell}\; I_{loc}(r,\varphi)$$

transfer function G

$$g = \frac{E}{E_{loc}}$$

$$\mu_e = \cos\theta_e$$

$$\ell = \frac{dS_o}{dS_{loc}^{\perp}}$$

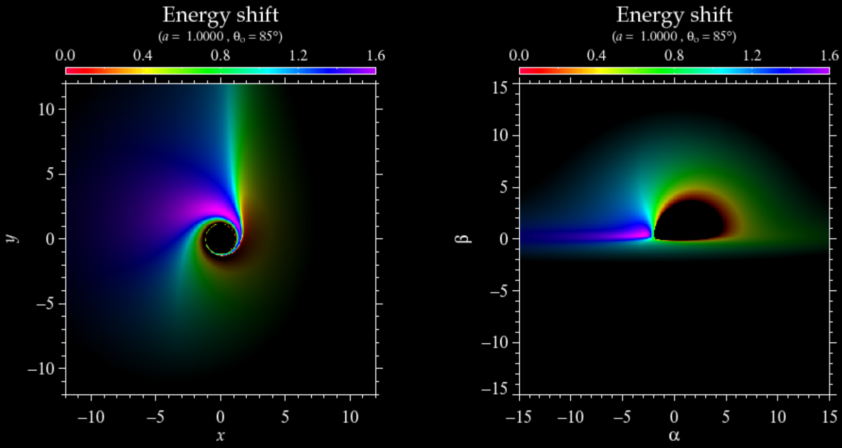
$\gamma = 4$ for $\frac{dE}{dt\,d\Omega\,dS}$

$\gamma = 3$ for $\frac{dE}{dt\,d\Omega\,dS\,d\nu}$ or $\frac{dN}{dt\,d\Omega\,dS}$

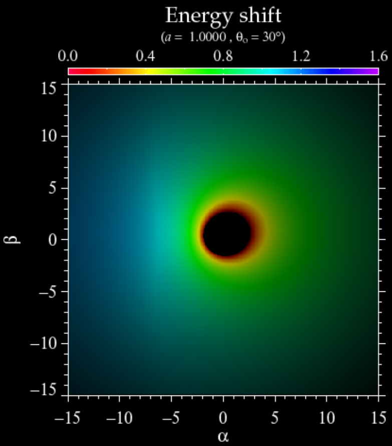
$\gamma = 2$ for $\frac{dN}{dt\,d\Omega\,dS\,d\nu}$

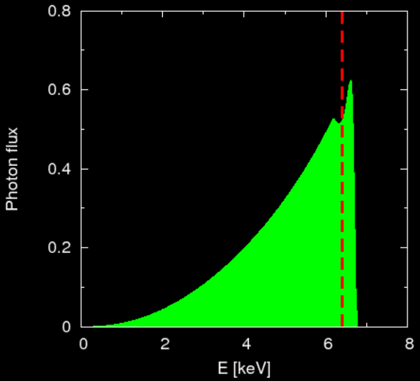
$$\frac{d\alpha\, d\beta}{r\, dr\, d\varphi} = \frac{dS_o}{dS_{loc}^{\perp}} \times \frac{dS_{loc}^{\perp}}{dS_{loc}} \times \frac{dS_{loc}}{dS} = \frac{\ell\,\mu_e}{g}$$

Energy shift and amplification



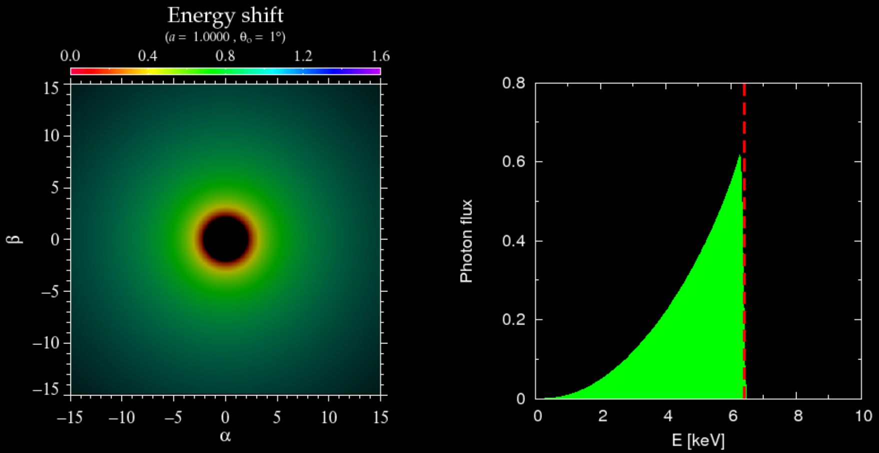
Dependence on spin





$$F_{\text{loc}} = r^{-3} \times \delta(E - E_{\text{rest}})$$

Dependence on inclination



$$F_{\text{loc}} = r^{-3} \times \delta(E - E_{\text{rest}})$$

Historical note on relativistic Fe Kα line

Spectrum from the black hole inner accretion disc:

Cunningham C. T. (1975) *The effects of redshifts and focusing on the spectrum of an accretion disk around a Kerr black hole*, ApJ, **202**, 788

Relativistic broad line profiles:

Fabian A. C., Rees M. J., Stella L. & White N. E. (1989) *X-ray fluorescence from the inner disc in Cygnus X-1*, MNRAS, **238**, 729

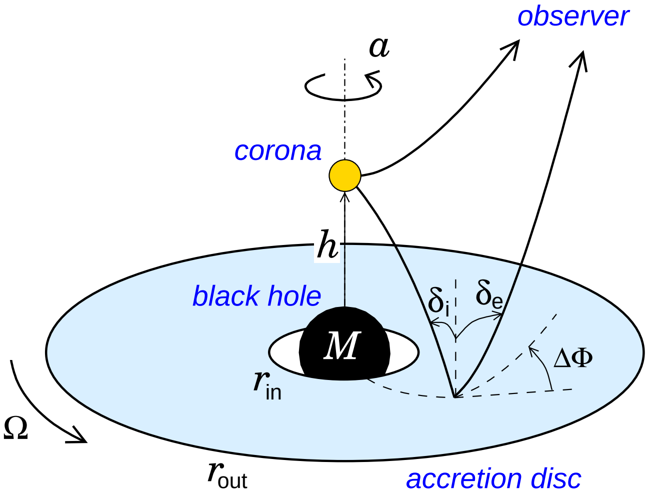
Observations:

Tanaka Y. et al. (1995) *Gravitationally redshifted emission implying an accretion disk and massive black hole in the active galaxy MCG-6-30-15*, Nature, **375**, 659

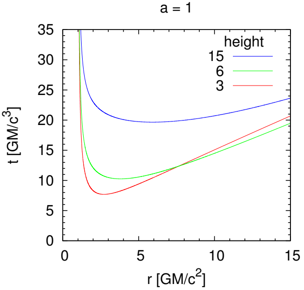
Fabian A. C. et al. (2002) *A long hard look at MCG-6-30-15 with XMM-Newton*, MNRAS, **335**, L1

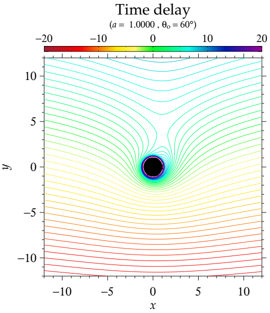
Miller J. M. (2007) *Relativistic X-Ray Lines from the Inner Accretion Disks Around Black Holes*, Annual Review of Astronomy & Astrophysics, **45**, 441

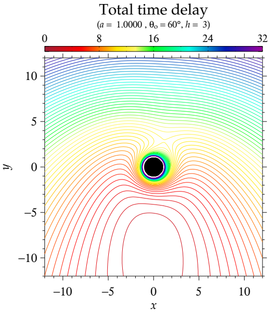
Scheme of the lamp-post geometry



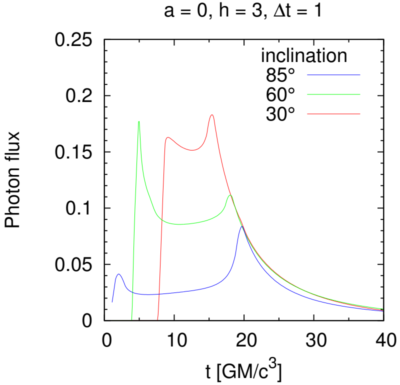
Time delay


a = 1
height
15
6
3
t [GM/c^3]
r [GM/c^2]

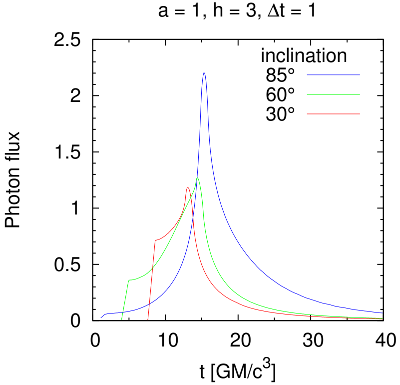

Time delay
(a = 1.0000 , θo = 60°)
y
x


Total time delay
(a = 1.0000 , θo = 60°, h = 3)
y
x

Light curve

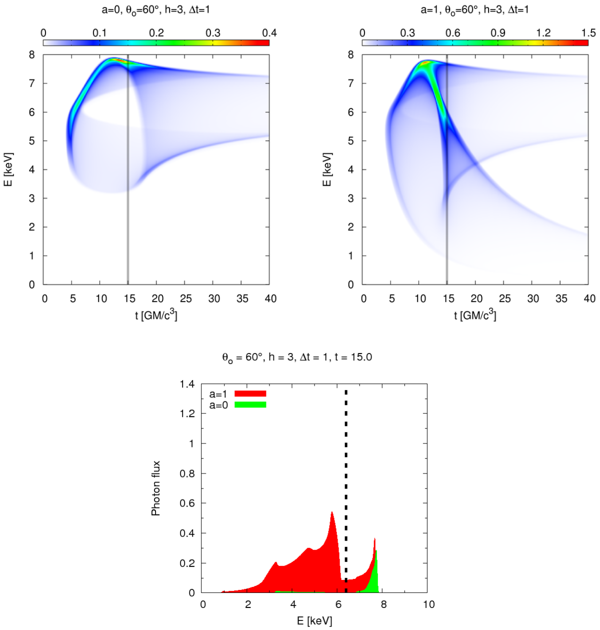


a = 0, h = 3, Δt = 1
Photon flux
t [GM/c^3]
inclination
85°
60°
30°
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40



a = 1, h = 3, Δt = 1
Photon flux
t [GM/c^3]
inclination
85°
60°
30°
2.5
2
1.5
1
0.5
0
0
10
20
30
40

Dynamic spectrum



Transfer function for reverberation

Stationary emission from the accretion disc:

$$F(E) = \int r\,dr\,d\varphi\,G(r,\varphi)\,F_{\mathrm{l}}(r,\varphi,E/g)$$

Response to the on-axis primary emission:

$$F(E,t) = \int dt' \int r\,dr\,d\varphi\,G(r,\varphi)\times$$

$$N_{\mathrm{p}}(t')\,N_{\mathrm{inc}}(r)\,M(r,\varphi,E/g,t'+t_{\mathrm{pd}})\,\delta([t-t_{\mathrm{do}}]-[t'+t_{\mathrm{pd}}])$$

Line reverberation:

$$F(E,t) = \int dt'\,N_{\mathrm{p}}(t') \int r\,dr\,d\varphi\,\Psi_0(r,\varphi)\,\delta(E-gE_{\mathrm{rest}})\,\delta([t-t']-\underbrace{[t_{\mathrm{pd}}+t_{\mathrm{do}}]}_{\Delta t})$$

$$G = g\,\mu_{\mathrm{e}}\,\ell$$

$$g = \frac{E}{E_{\mathrm{l}}}$$

$$\mu_{\mathrm{e}} = \cos\delta_{\mathrm{e}}$$

$$\ell = \frac{dS_{\mathrm{o}}}{dS_{\mathrm{l}}^{\perp}}$$

$$N_{\mathrm{inc}} = g_{\mathrm{inc}}^{\Gamma}\,\frac{d\Omega_{\mathrm{p}}}{dS_{\mathrm{inc}}}$$

$$\Psi_0 = g\,G\,N_{\mathrm{inc}}\,M$$

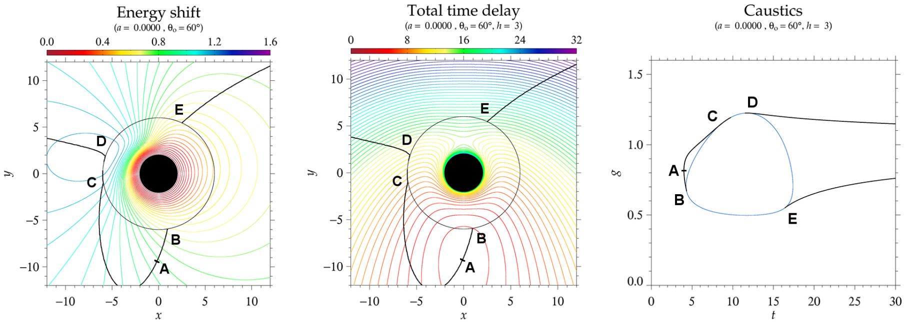
Transfer function $\rightarrow$ response to a flare $[N_{\mathrm{p}}(t') = \delta(t')]$:

$$\Psi(E,t) = \sum_{\substack{g=E/E_{\mathrm{rest}} \\ t_{\mathrm{pd}}+t_{\mathrm{do}}=t}} \Psi_0\,\frac{r}{E_{\mathrm{rest}}}\left|\frac{\partial(g,\Delta t)}{\partial(r,\varphi)}\right|^{-1}$$

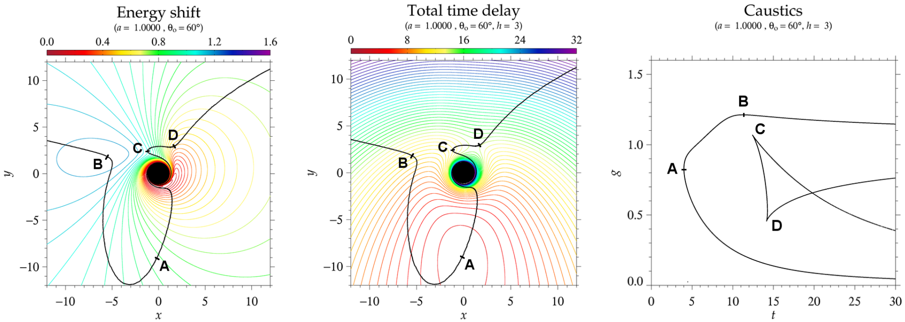
$$F(E,t) = \int dt'\,N_{\mathrm{p}}(t')\,\Psi(E,t-t')$$

$$\frac{\partial(g,\Delta t)}{\partial(r,\varphi)} = \frac{\partial g}{\partial r}\frac{\partial(\Delta t)}{\partial\varphi} - \frac{\partial g}{\partial\varphi}\frac{\partial(\Delta t)}{\partial r} \neq 0 \qquad \Rightarrow \qquad \nabla g \nparallel \nabla(\Delta t)$$

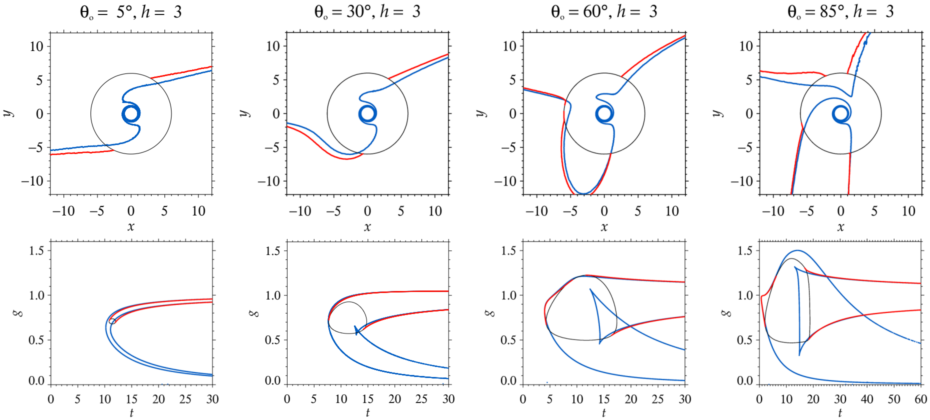
Caustics – Schwarzschild case



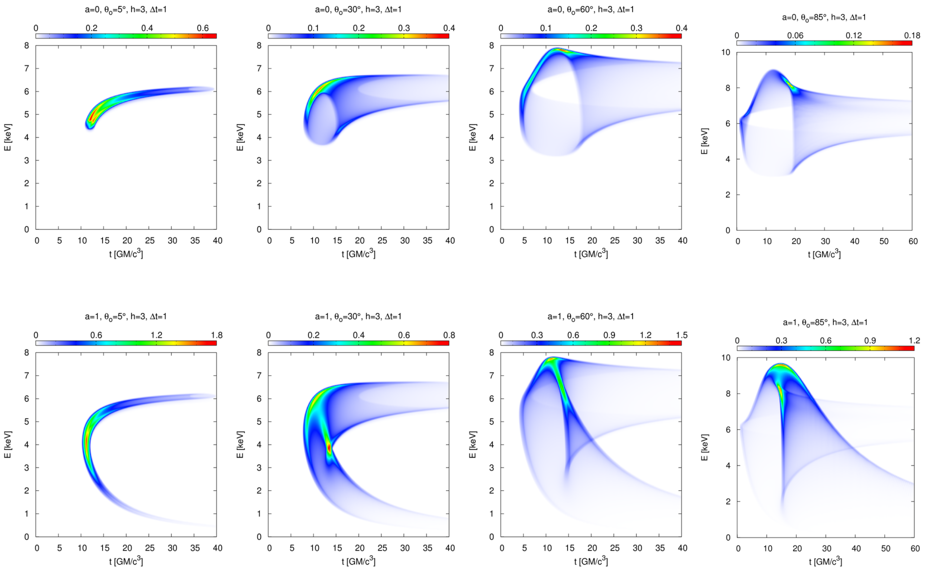
Caustics – extreme Kerr case



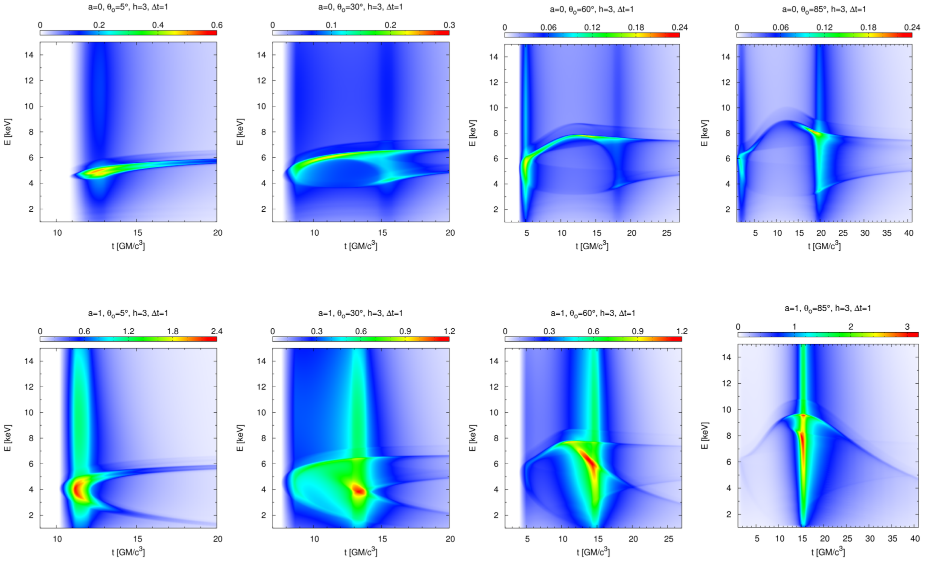
Caustics



Dynamic spectrum – narrow spectral line



Dynamic spectrum – the whole reflection



Dynamic spectrum – ionised disc

$$E^2 \times F(E)$$