Binary black hole embedded in an external magnetic field as site of particle acceleration





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Black holes can form a compact binary where the orbital motion reaches a significant fraction of velocity of light. This can make the components of the black hole binary system moving at high speed through the magnetic field of external origin. We demonstrate that a black hole transiting across a large-scale magnetic filament induces an electric component, which then accelerates electrically charged particles in the inner magnetosphere. We identify the location of the acceleration point near the ergosphere boundary of the black hole as a function of its linear velocity (translatory boost). Members of the binary black-hole system in a tight orbit around each other should exhibit this effect at greater efficiency because they can reach high linear velocity in the orbit.

Introduction

Filaments of enhance magnetic intensity appear to exist in the Milky Ways central region (Sagittarius A*). Also in connection with the jet formation, an organized component of the magnetic field arises on length-scales exceeding the gravitational radius, as suggested in the case of M87 galaxy [1]. In case of a compact binary system, its components can reach a high translational velocity, which leads to the induction of an intense electric field.

In this context we construct and study the solutions describing a uniform magnetic field under the influence of a rotating (Kerr) black hole. Since the Kerr metric is asymptotically flat, this electromagnetic field simplifies to the original homogenous magnetic field in the asymptotic region, as given originally by Wald [2] for the special case of perfect alignment with the symmetry axis. We employ a more general solution [3] with arbitrary inclination to construct the electromagnetic structure around the Kerr black hole that drifts through the external magnetic field at arbitrary velocity and in arbitrary direction [4].

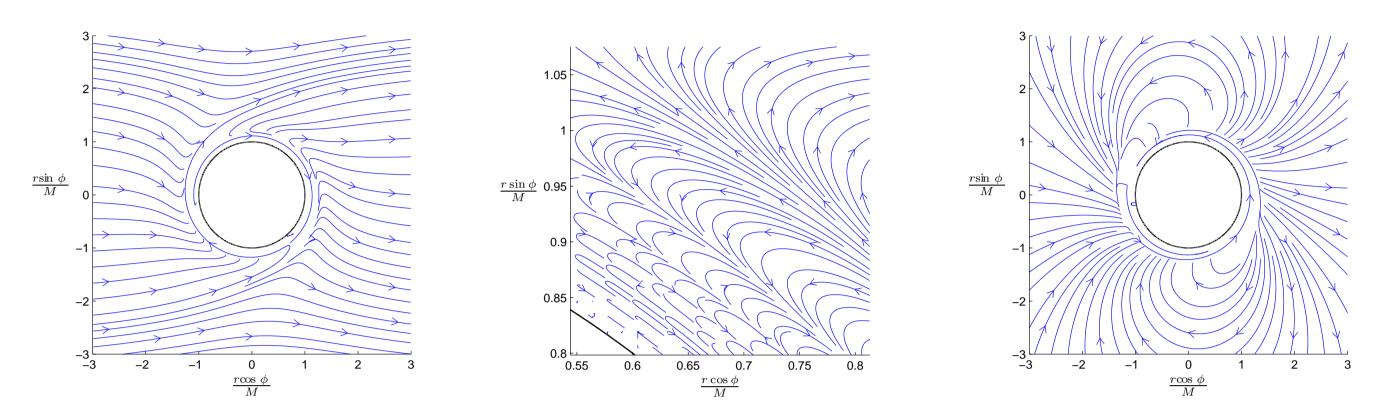


Fig. 1: A co-rotating observer around a nondrifting extreme Kerr black hole measures the magnetic field (as shown in the left and the middle panel) and an equatorial projection of the electric field (in the right panel).

Structure of electromagnetic field: magnetic null points and layers

We start with $F_{\mu\nu}$ describing the test field with asymptotic form of a general (ie. not necessarily parallel) uniform magnetic field given in [3]. Due to the axial symmetry of Kerr space-time only two components of asymptotic field were considered in that paper without any loss of generality (asymptotic components B_0 (parallel) and B_1 (equatorial) to be specific). We split components of EM tensor (eq. (A3) of [3]) denoting $B_x \equiv B_1$, $B_z \equiv B_0$ into two parts $F_{\mu\nu}^{B_x}$ and $F_{\mu\nu}^{B_y}$ according to the asymptotic component.

As we shall introduce a drift of the black hole in the general direction we lose axial symmetry and need to consider all spatial components of the asymptotic magnetic field. We obtain $F_{\mu\nu}^{B_y}$ (which may only appear due to nonzero drift) by rotating $F_{\mu\nu}^{B_x}$ along the z-axis by angle $\frac{\pi}{2}$ - i.e. $F_{\mu\nu}^{B_y} = F_{\mu\nu}^{B_x} \left(\phi \to \phi - \frac{\pi}{2}, B_x \to B_y \right)$.

Since the drift shall induce uniform electric field in the asymptotic region we need to have appropriate $F_{\mu\nu}^{E_{x,y,z}}$ handy. We get them easily by performing dual transformation of $F_{\mu\nu}^{B_{x,y,z}}$. Dual transformation is carried out as follows:

$$^*F_{\alpha\beta} = \frac{1}{2}F^{\mu\nu}\varepsilon_{\mu\nu\alpha\beta},$$

where $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor whose components are given as [5]:

$$\varepsilon_{\mu\nu\alpha\beta} = \sqrt{-\det||g_{\sigma\omega}||}[\mu\nu\alpha\beta] \equiv \sqrt{-g}[\mu\nu\alpha\beta],$$

with $[\mu\nu\alpha\beta]$ denoting completely antisymmetric symbol. Determinant of the Kerr metric in Boyer-Lindquist coordinates t,r,θ and ϕ is $g = g_{tt}g_{rr}g_{\theta\theta}g_{\phi\phi} - g_{\phi t}^2g_{rr}g_{\theta\theta} = -\sin^2\theta \Sigma^2$.

Performing the dual transformation we immediately obtain EM tensors with desired asymptotics of uniform electric field:

$$F_{\mu\nu}^{E_{x,y,z}} = {}^*F_{\mu\nu}^{B_{x,y,z}}(B_{x,y,z} \to -E_{x,y,z}).$$

Now we are fully equipped to construct any asymptotically uniform test field on the Kerr background just by linear superposing of the above EM tensors. As we are concerned in constructing $F_{\mu\nu}$ which describes the test field around the black hole drifting through asymptotically uniform magnetic field in general direction, we shall employ Lorentz transformation to find the correct asymptotic components of such a field. Once obtained we just use them to replace the original "nondrifting" quantities E_x, E_y, E_z, B_x, B_y and B_z .

In matrix formalism we perform the transformation as follows:

$$||F_{\mu\nu}^{\text{asymptotic}}|| = ||\Lambda_{\mu}^{\mu'}||^{t} ||F_{\mu'\nu'}^{\text{asymptotic}}|| ||\Lambda_{\nu}^{\nu'}|| = ||\Lambda_{\mu}^{\mu'}|| ||F_{\mu'\nu'}^{\text{asymptotic}}|| ||\Lambda_{\nu}^{\nu'}||, \tag{1}$$

where $||\Lambda^{\mu'}_{\mu}||$ is the matrix of general Lorentz transformation [6].

Final step of the derivation is thus substitution of "nondrifting" quantities $E_{x,y,z}$ and $B_{x,y,z}$ in the tensors $F_{\mu\nu}^{E_{x,y,z}}$ and $F_{\mu\nu}^{B_{x,y,z}}$ by Lorentz transformed values from eq. (1) and superposing the components to acquire general EM tensor describing the field around the Kerr black hole drifting through asymptotically uniform magnetic field of general orientation:

$$F_{\mu\nu} = F_{\mu\nu}^{E_x}(E_x \to -v_y \gamma B_z) + F_{\mu\nu}^{E_y}(E_y \to \gamma (v_x B_z - v_z B_x)) + F_{\mu\nu}^{E_z}(E_z \to v_y \gamma B_x) + F_{\mu\nu}^{B_x}(B_x \to \gamma B_x - v_x D) + F_{\mu\nu}^{B_y}(B_y \to -v_y D) + F_{\mu\nu}^{B_z}(B_z \to \gamma B_z - v_z D),$$

where v_x , v_y , v_z are components of the drift speed, γ is it's Lorentz factor and $D \equiv \frac{\gamma^2}{\gamma+1}(v_z B_z + v_x B_x)$.

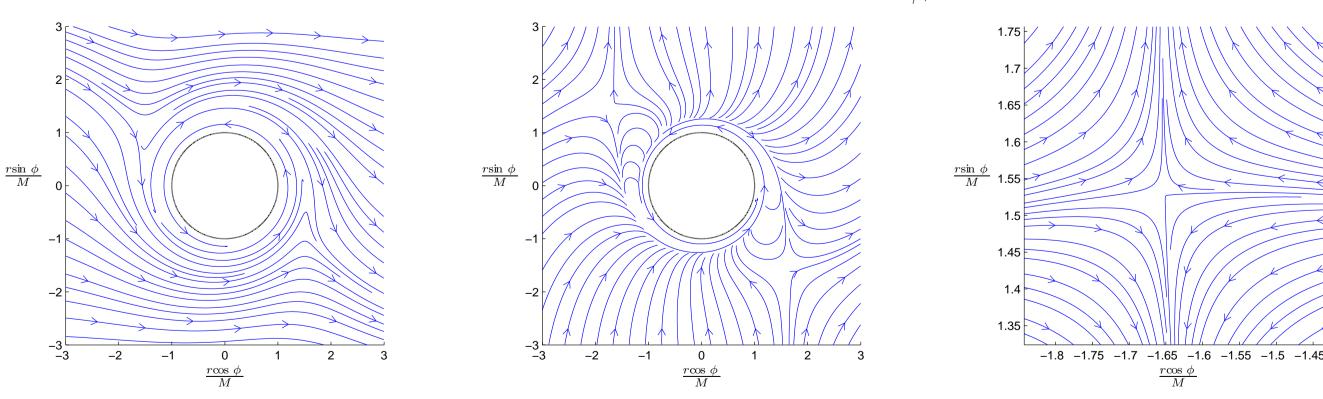


Fig. 2: Counter-rotating observer around a nondrifting extreme Kerr black hole measures magnetic field (in the first panel) and the equatorial projection of the electric field (in the second panel). In the third panel the vicinity of the null point has been enlarged by zooming into its immediate neighborhood.

Conclusions

We derived an explicit example of the electromagnetic test-field structure around a drifting Kerr black hole. This provides us with a method to model the magnetic field structure in the vicinity of a component of a binary black hole system in vacuum. Components of the electromagnetic tensor $F_{\mu\nu}$ were given and the structure of the resulting fields has been plotted in the special case of and asymptotically uniform magnetic field perpendicular to the symmetry axis. Magnetic null points can be found near the ergosphere, where the reconnection can occur and the particle acceleration by an induced electric component becomes efficient.

Lines of electric and magnetic field

We shall make just a brief qualitative overview of the possible structures of the lines of force of magnetic and electric intensities felt by the above specified observer which may be either co-rotating or counter-rotating with the background geometry. In all figures we show a portion of the equatorial plane. The circle signifies the location of an outer event horizon.

Original asymptotically uniform magnetic field is restricted to be perpendicular to the symmetry axis (i.e. it lies in the equatorial plane). Without any loss of generality we align it with the horizontal axis of presented figures (only nonzero asymptotical component is thus B_x). Since the impact of the rotation of the black hole upon the structure of the field lines is most apparent for extreme Kerr black hole (a = M) we confine ourselves to this case from now on. Marginally stable orbit $r_{\rm ms}$ thus coincides with the horizon at r = M for prograde orbits while it lies at $r_{\rm ms} = 9M$ for retrograde orbits.

First we present the field lines around a nondrifting (stationary) black hole; then the linear drift is introduced to bring the black hole into translatory motion. The boost induces an additional electric component.

Natural definition of the lines of force (of magnetic and electric fields), as measured by given observer equipped with the orthonormal tetrad $e^{\mu}_{(\alpha)}$, is their identification with the lines along which magnetic/electric charge connected to this observer would accelerate due to the presence of the EM field F^{μ}_{ν} . Coordinate components of the magnetic (electric) field intensities are determined by (coordinate components of) the Lorentz force felt by the unit magnetic (electric) charge [7]:

$$B^{\mu} = {}^*F^{\mu}_{\ \nu}u^{\nu}, \quad E^{\mu} = F^{\mu}_{\ \nu}u^{\nu},$$

where u^{ν} represents a 4-velocity of the charge.

Tetrad components of the vector field determining desired lines of force are given as the spatial part of the projection onto $e^{\mu}_{(\alpha)}$:

$$B^{(i)} = B_{(i)} = e^{(i)}_{\mu} * F^{\mu}_{\nu} u^{\nu}, \quad E^{(i)} = E_{(i)} = e^{(i)}_{\mu} F^{\mu}_{\nu} u^{\nu},$$

where $e^{(\alpha)}_{\mu}$ are 1-forms dual to the tetrad vectors $e^{\mu}_{(\alpha)}$. Lowering/rising spatial tetrad indices doesn't matter since the tetrad is supposed to be orthonormal – $g_{(\mu)(\nu)} = \eta_{(\mu)(\nu)}$.

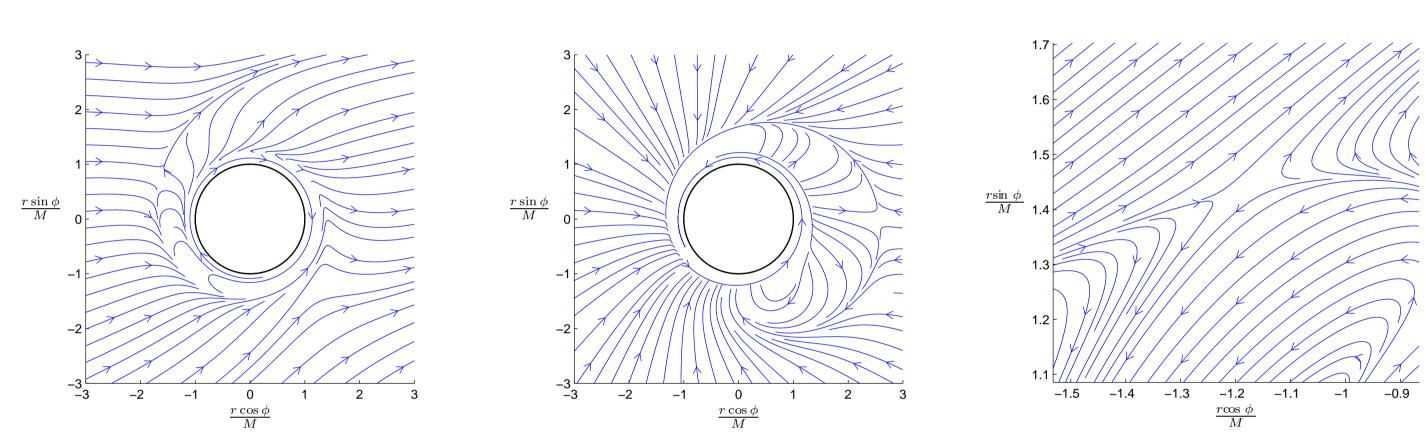


Fig. 3: A co-rotating observer around a drifting ($v_x = 0.5c$ and $v_y = -0.7c$) extreme Kerr black hole measures magnetic field (left panel) and the equatorial projection of the electric field (middle panel). A counter-rotating observer around a drifting ($v_x = 0.5 c$ and $v_y = -0.7 c$) extreme Kerr black hole measures the magnetic field and detects a null point (zoomed in the right panel).

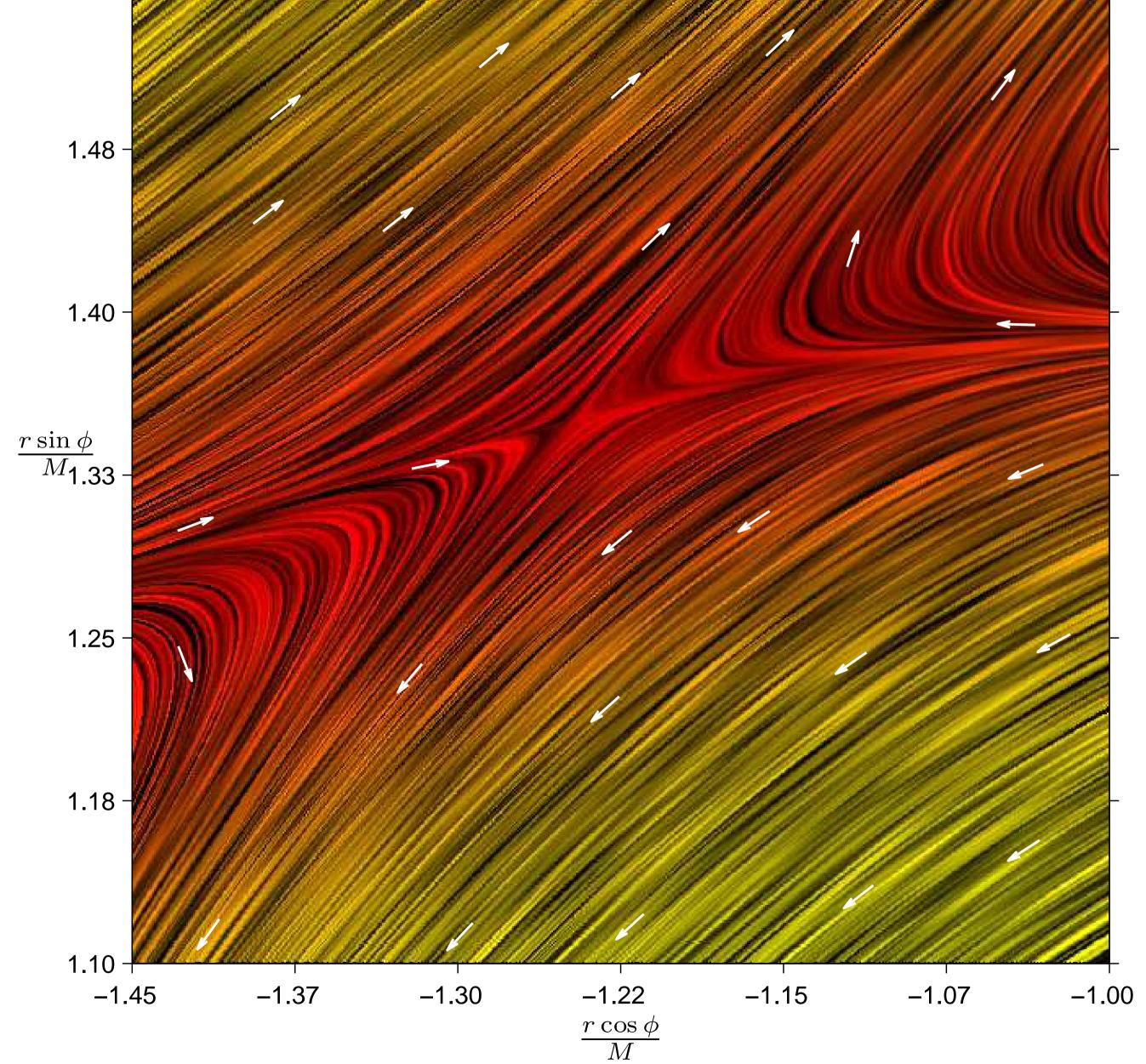


Fig. 4: LIC method has been employed to visualize the vicinity of the magnetic null point revealed in Fig. 3.

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