Investigating the Relativistic Motion of the Stars near the Super-Massive Black Hole in the Galactic Center

Stellar Dynamics in Galactic Nuclei -Workshop 2017

Nov. 29 – Dec. 1, Princeton, NJ, USA

SEVENTH FRAMEWORK PROGRAMME

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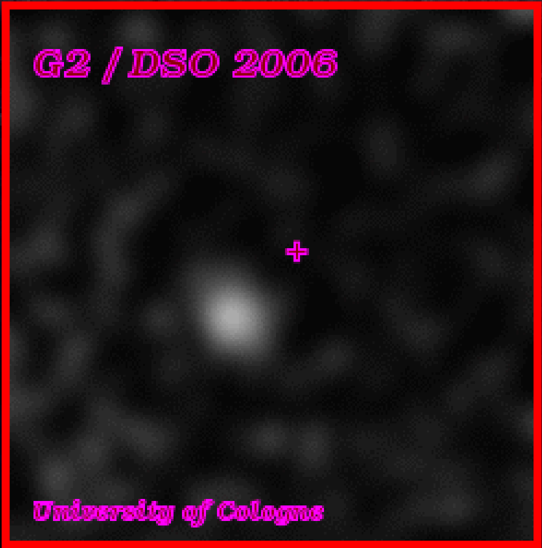
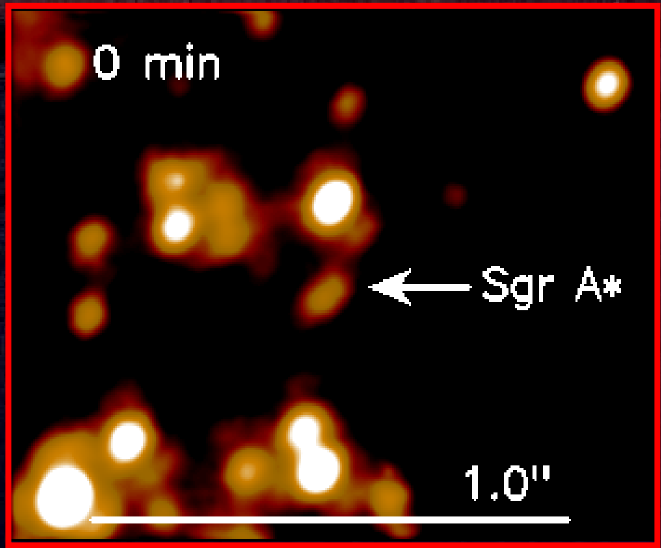
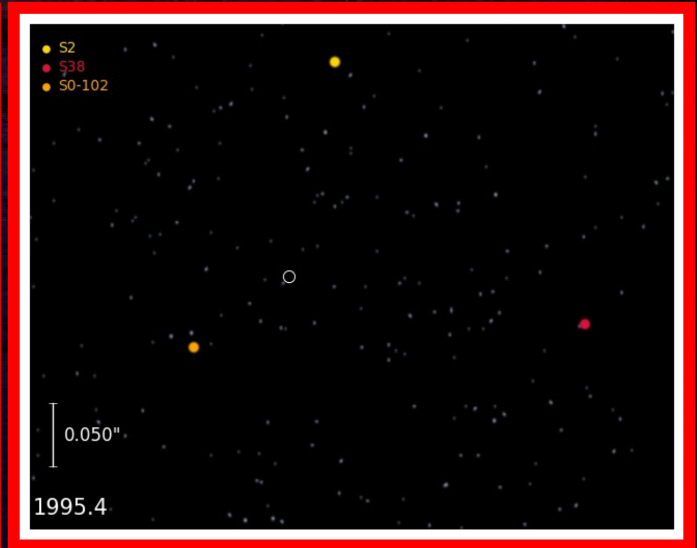
STRONGGRAVITY

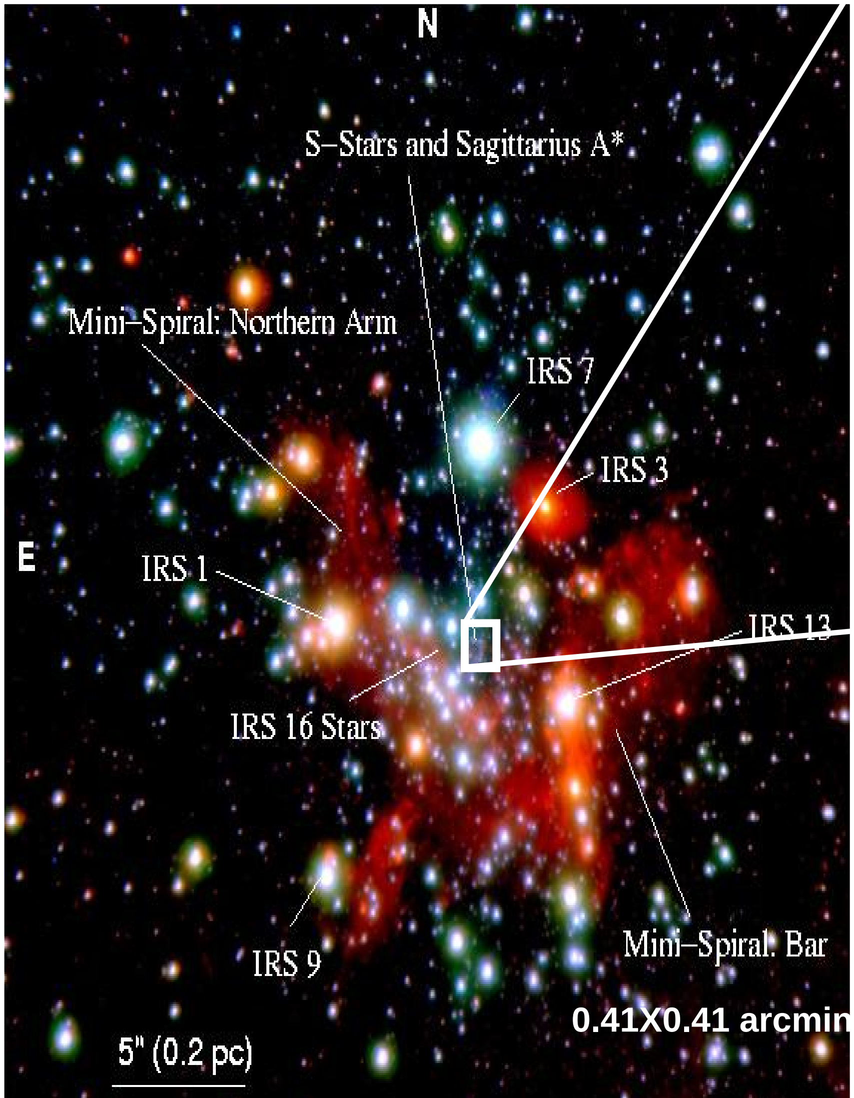
EU FP7-SPACE research project 312789 2013 - 2017

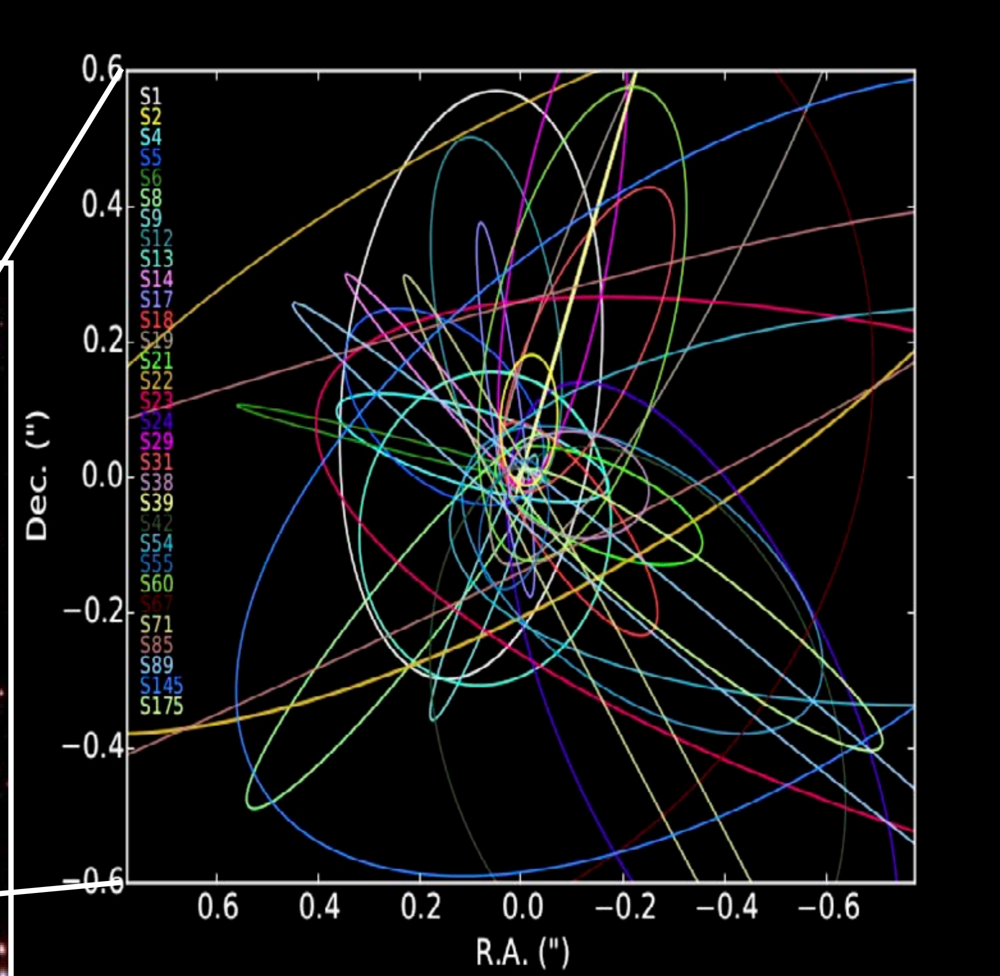






The S-cluster





Orbits of 31 stars:

- **23 orbits: aegroup**
- **8 orbits: Gillessen et al. (2017)**

Eckart & Genzel (1996/1997): First proper motions

Credit: NACO/ESO/University of Cologne

Outlook

- Investigate the gravitational potential parameters of Sgr A* including the **mass** of and the **distance** to it through stellar motion

- Develop a **new and practical method to investigate the GR effects** on the proper motion of the stars closest to Sgr A*

- Generate **representative stellar orbits** using a first-order **post-Newtonian approximation** with a broad range of periapse distance

- **Apply the results to data on S2 star**

M. Parsa, A. Eckart, B. Shahzamanian, V. Karas, M. Zajaček, J. A. Zensus, and C. Straubmeier, **2017 ApJ 845, 1**

NIR Observations

Site: Paranal, Chile
Telescope: Very Large Telescope
Instrument: NACO = NAOS+CONICA
Wavelength Coverage: 1-5 μm

- K_s-band: 2.18 μm
- S13 camera:
  FoV: 14”X14”
  Scale: 13.3 mas/pix
- S27 camera:
  FoV: 28”X28”
  Scale: 27 mas/pix

Y. Beletsky (LCO)/ESO

Data Analysis

- Data reduction:
  1. flat-fielding
  2. sky subtraction
  3. bad pixel correction
- S13 images: Lucy-Richardson deconvolution, resolving the S-stars
- S27 images: 8 SiO maser stars: IRS9, IRS10EE, IRS12N, IRS15NE, IRS17, IRS19NW, IRS28 and SiO-15 (Reid et al. 2007)
- Short orbital period data covering large portion of the orbit
- Only data with SgrA* flaring to ensure registration

S2:

K_s = 14.2
Period = 16.2 yr
33 measurements

S38:

K_s = 17
Period = 18.6 yr
29 measurements

S0-102:

(Meyer et al. 2012)
also known as S55
K_s = 17.1
Period = 12 yr
25 measurements

2002 - 2015

Registration

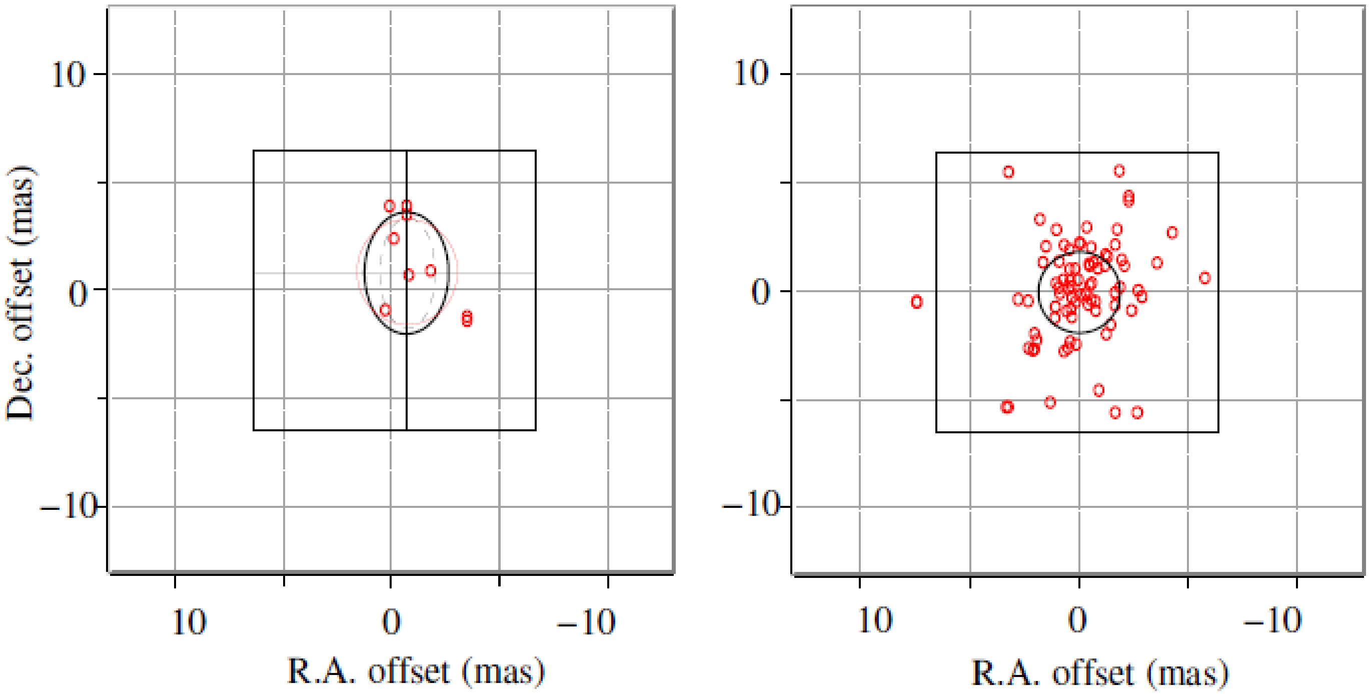


Figure 1. Left: Single epoch statistics for the offset between the infrared and radio positions of Sgr A*. The uncertainties for the R.A. and Dec.: With respect to the median offset the zero offset point is well included in the median deviation: 1.8 mas × 0.9 mas (thin red ellipse); the standard deviation: 2.0 mas × 1.4 mas (thick black ellipse); the equivalent geometrical mean: 1.7 mas (black dashed ellipse); Right: Single epoch statistics for all maser sources well centered on the zero offset point. The standard deviation is 1.8 mas (black circle).

Data that contain SgrA* flaring only

Srg A* drift

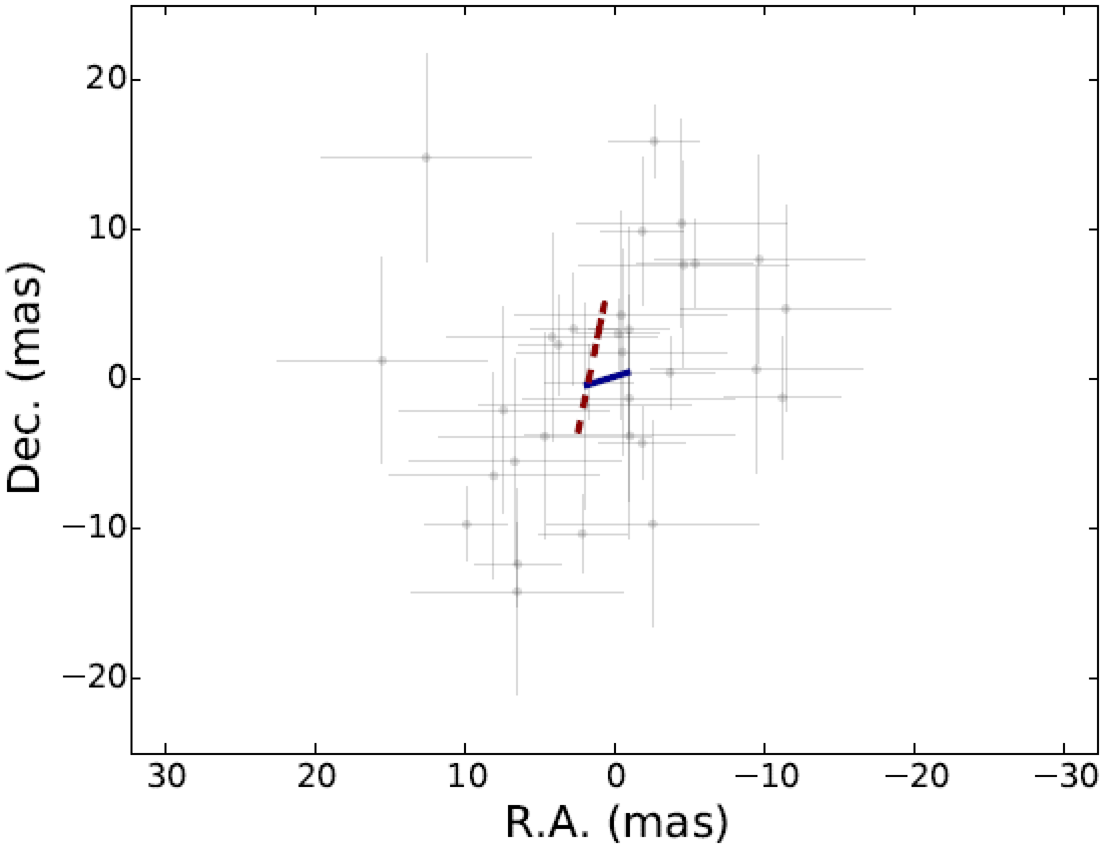


Figure 2. Linear motion fit to the Sgr A* NIR counterpart data (derived from Newtonian orbit fitting to all three stars) after applying the correction described in the text in this study (solid blue) compared to a recent study (dashed red, Gillessen et al. (2009b)). The data point with a cross indicating their uncertainties are the positions we derived for the IR counterpart of Sgr A*.

Checking against S38 data for rotation (included in MCMC)

Models

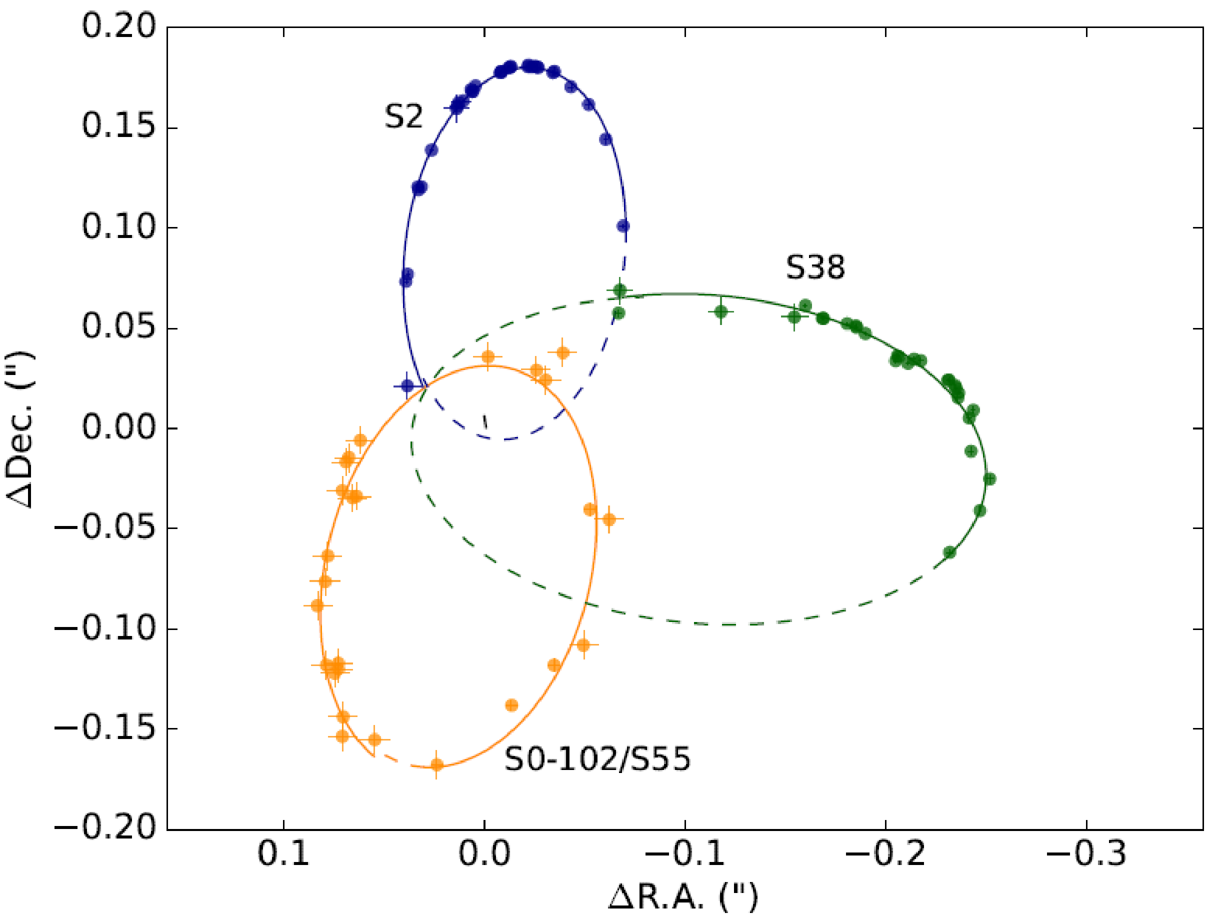
- Newtonian (Keplerian) Model: 6 orbital elements
- Post-Newtonian (PN) Model:
    - Approximate solution to Einstein's equations
    - Expansions of a small parameter: v/c
- Einstein-Infeld-Hoffmann (Einstein et al. 1938) equation of motion:

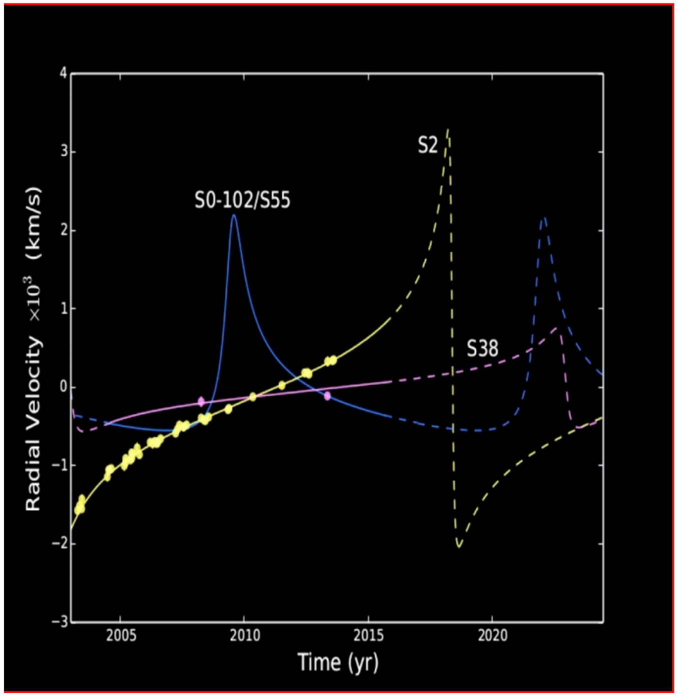
$$\frac{d\boldsymbol{v}_\star}{dt} = -\frac{GM_{BH}}{c^2 r_\star^3}\left\{\boldsymbol{r}_\star\left[c^2 + v_\star^2 + 2v_{BH}^2 - 4\left(\boldsymbol{v}_\star.\boldsymbol{v}_{BH}\right)\right.\right.$$
$$\left.\left.-\frac{3}{2r_\star^2}\left(\boldsymbol{r}_\star.\boldsymbol{v}_{BH}\right)^2 - 4\frac{GM_{BH}}{r_\star}\right] - \left[\boldsymbol{r}_\star.\left(4\boldsymbol{v}_\star - 3\boldsymbol{v}_{BH}\right)\right]\left(\boldsymbol{v}_\star - \boldsymbol{v}_{BH}\right)\right\}$$

- or for negligible proper motion of the SMBH (Rubilar & Eckart 2001):

$$\frac{d\boldsymbol{v}_\star}{dt} = -\frac{GM_{BH}}{c^2 r_\star^3}\left[\boldsymbol{r}_\star\left(c^2 - 4\frac{GM_{BH}}{r_\star} + v_\star^2\right) - 4\boldsymbol{v}_\star\left(\boldsymbol{v}_\star.\boldsymbol{r}_\star\right)\right]$$

Relativistic and non-relativistic fits to the data



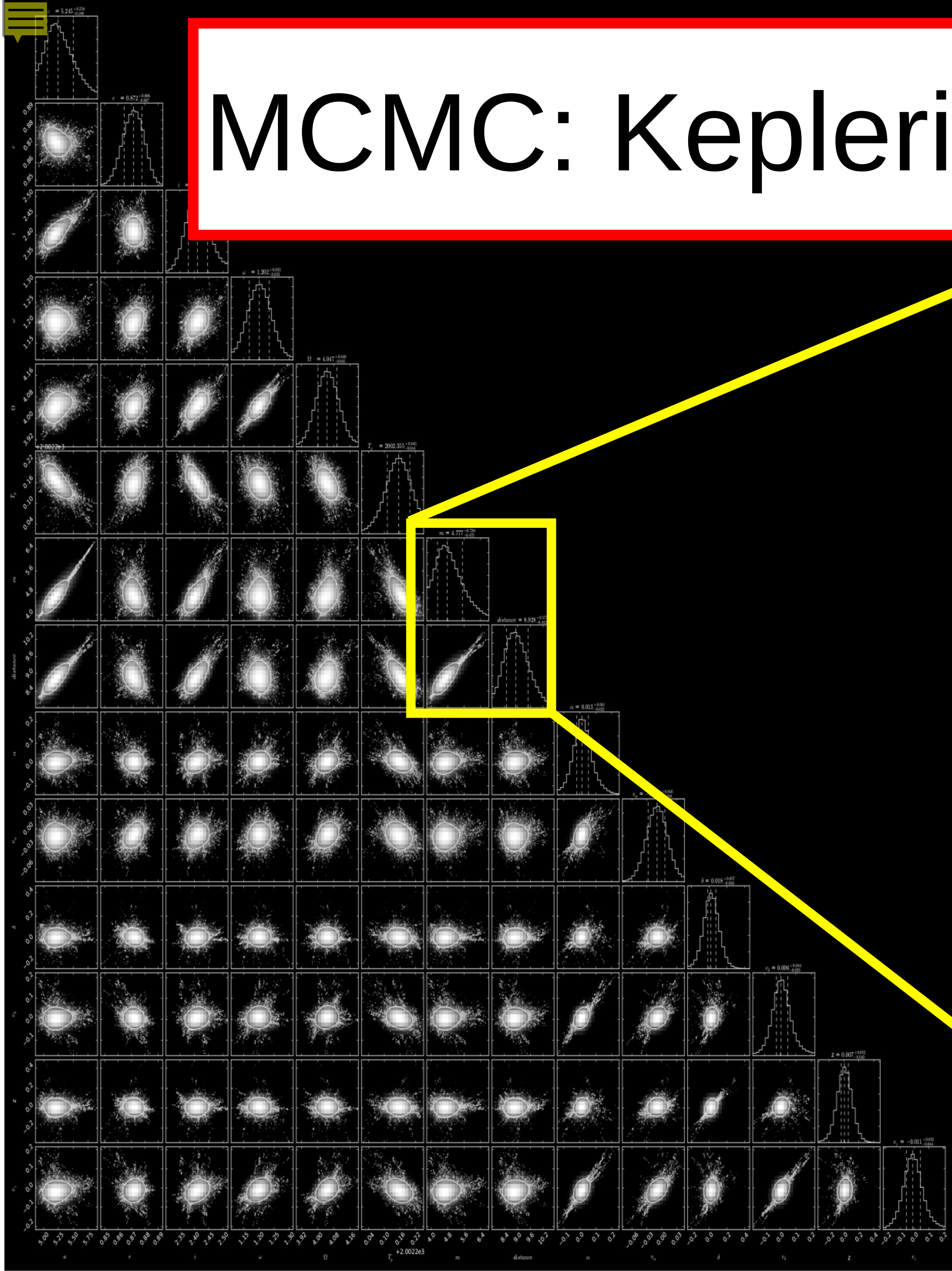


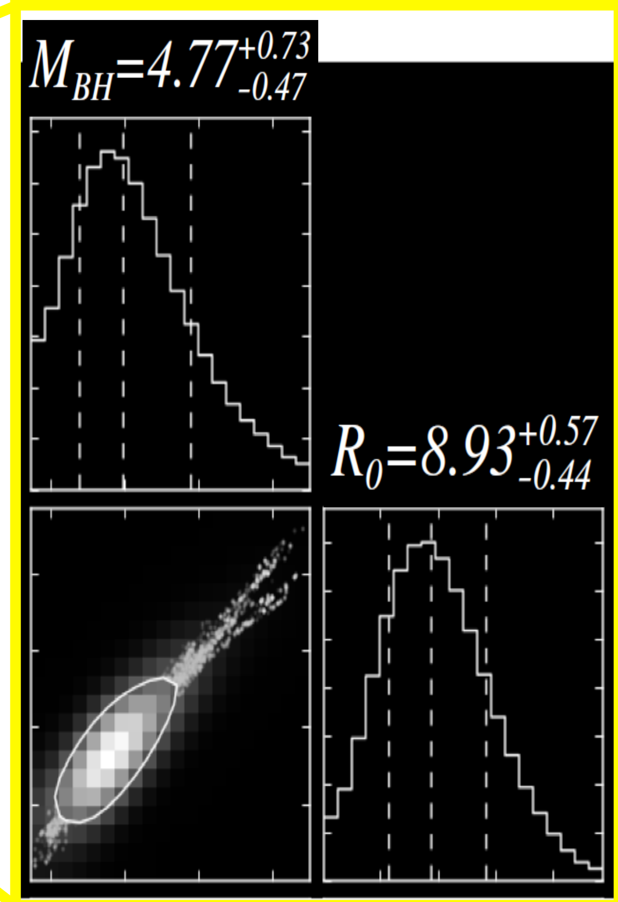
We modeled the stellar orbits in by integrating the equations using the ***4th*** **order Runge-Kutta method** with up to twelve initial parameters, respectively (i.e. the positions and velocities in 3 dimensions).

Parsa et al. (2017)

In addition to the VLT data, we used published (not shown here) Keck positions by Boehle et al. (2016) in years 1995-2010 and radial velocities by Gillessen et al. (2009) Boehle et al. (2016)

MCMC: Keplerian Model - S2





Fitting Parameters:

- 6 Orbital Parameter/State Vectors
- 7 Gravitational Potential Parameters

Best MCMC Results

S2 periapse: 2018.51 +- 0.22 which is in July

- Model: Keplerin / Relativistic
- Data: S2 / S2 & S38 / S2 & S38 & S55
- Best Results: Keplerian Model - Three Stars

Black Hole Parameters

M_{BH} (10^6 $M_\odot$)	$4.15^{+0.09}_{-0.13}$
Distance (kpc)	$8.19^{+0.08}_{-0.11}$
α (mas)	$0.19^{+0.04}_{-0.04}$
δ (mas)	$-0.16^{+0.03}_{-0.41}$
v_α (mas/yr)	$-0.03^{+0.05}_{-0.06}$
v_δ (mas/yr)	$0.02^{+0.02}_{-0.03}$
v_z (mas/yr)	$0.70^{+1.47}_{-1.52}$

	S2	S38	S55
a (arcsec)	$0.126^{+0.001}_{-0.001}$	$0.140^{+0.007}_{-0.002}$	$0.109^{+0.002}_{-0.002}$
e	$0.884^{+0.002}_{-0.002}$	$0.818^{+0.005}_{-0.005}$	$0.75^{+0.01}_{-0.01}$
i (deg)	$136.78^{+0.36}_{-0.44}$	$166.22^{+3.1}_{-2.4}$	$147.7^{+1.6}_{-1.5}$
ω (deg)	$71.36^{+0.65}_{-0.84}$	$18.4^{+4.8}_{-5.8}$	$133.5^{+3.9}_{-3.6}$
Ω (deg)	$234.50^{+0.94}_{-1.09}$	$101.8^{+4.6}_{-5.6}$	$129.9^{+4.0}_{-4.2}$
T_p (yr)	$2002.32^{+0.02}_{-0.02}$	$2003.30^{+0.03}_{-0.04}$	$2009.31^{+0.03}_{-0.03}$

Relativistic Orbits of Stars

General Relativistic Effects

Effects:

Astrometric

Spectroscopic

Lower order effects: Transverse Doppler Shift, Gravitational Redshift (**Zucker et al. 2006**; Angélil et al. 2010; Zhang et al. 2015), Periapse Shift (proper motion**; Rubilar & Eckart 2001**: first discussion for GC), equivalent: effects on long half axis and ellipticity of the orbit **Parsa et al. 2017, Iorio 2017**).

Higher order effects: Frame-dragging (Lense-Thirring) (Iorio & Zhang 2017, Zhang & Iorio 2017) , Gravitational Lensing

Periapse shift has at least 3 major contributors

- In-plane precession:

  1. **Prograde relativistic**: general relativistic effect (mass and spin of the black hole)

  2. **Retrograde Newtonian**: presence of distributed mass, longer time scale at all distances

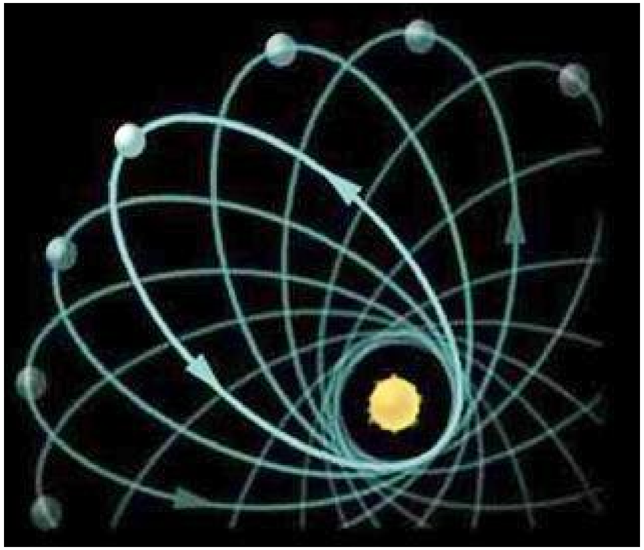
- Precession of orbital plane:

  1. Relativistic: spin (< 1 mpc)

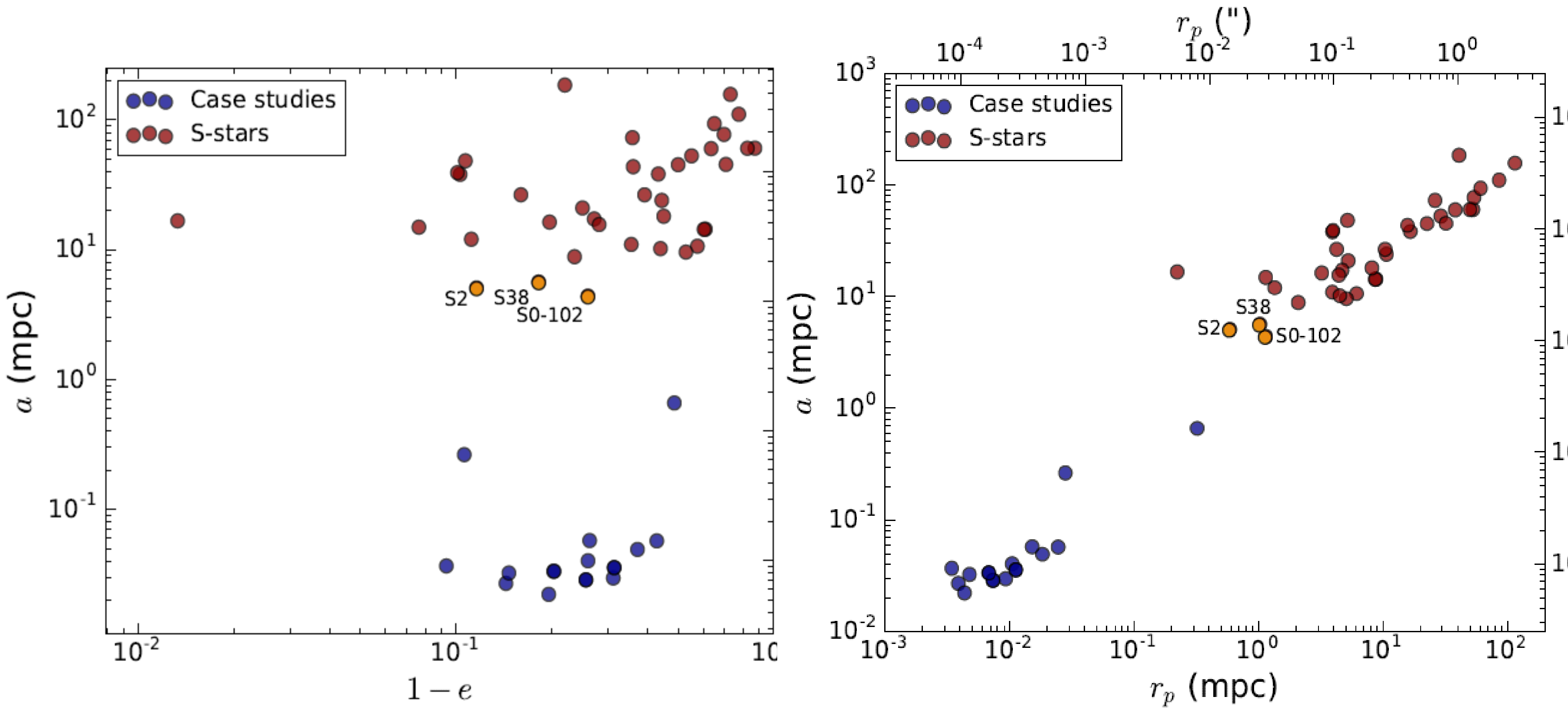
  2. Newtonian: **granularity of distributed mass**

     longer time scale at some distances

     (**Sabha et al. 2012**)



Distribution of Simulated Stars



Elements for S-stars, the three closest known S-stars, and simulated stars are shown. A resonable range of eccentricities and long axis between those of the S-stars and stars close to their tidal disruption limit are covered (~0.1mas).

Parsa et al. (2017)

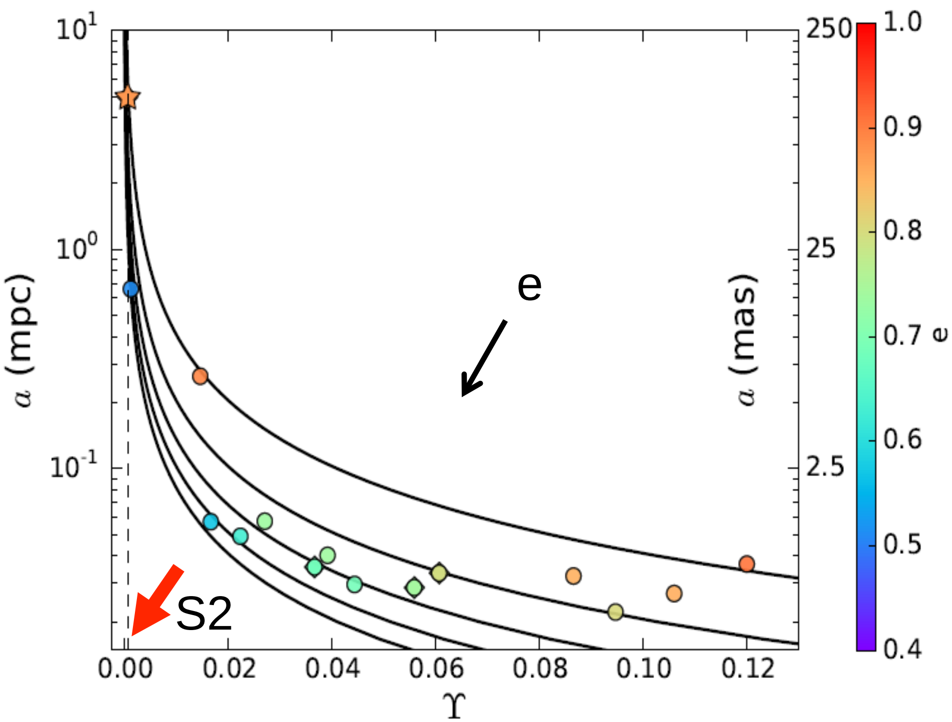
Relativistic Parameter at Periapse

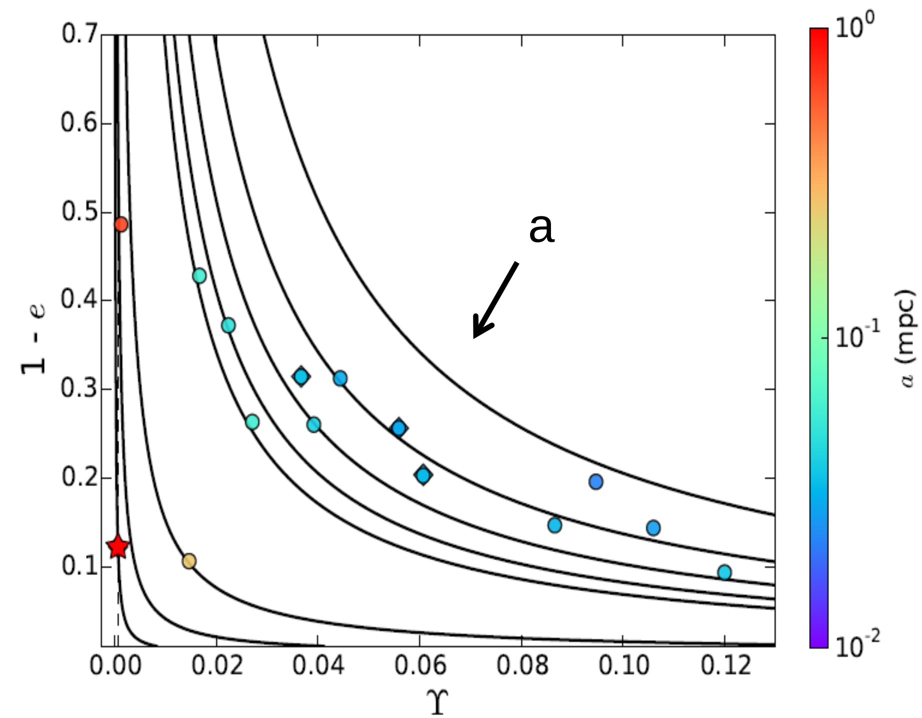
Relativistic Parameter Y:
Zucker et al. 2006

$$Y = \frac{r_s}{r_p}$$

r_s Schwarzschild radius
r_p periapse distance

$$\Delta\omega = \frac{3\pi}{1+e} Y$$

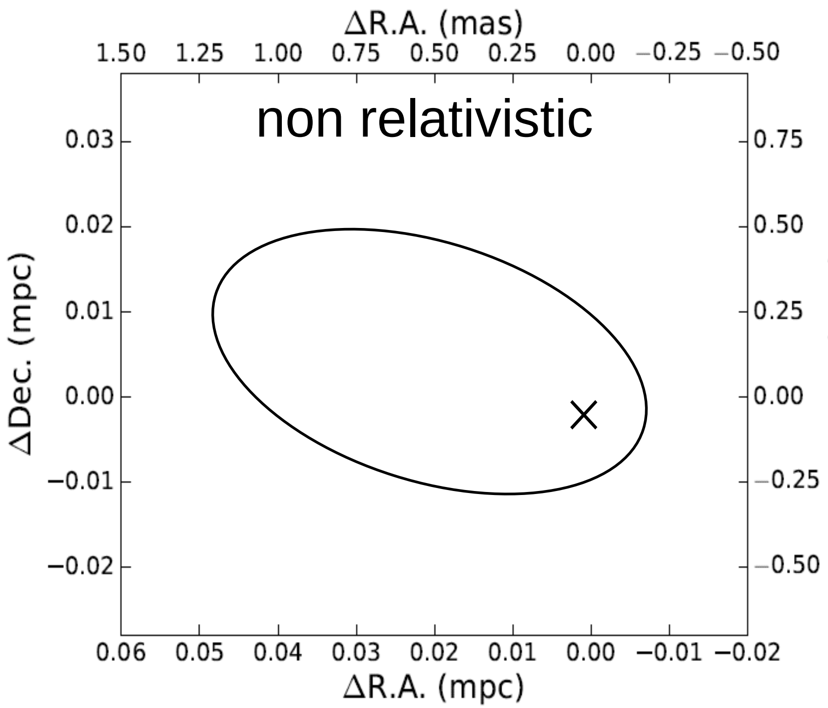




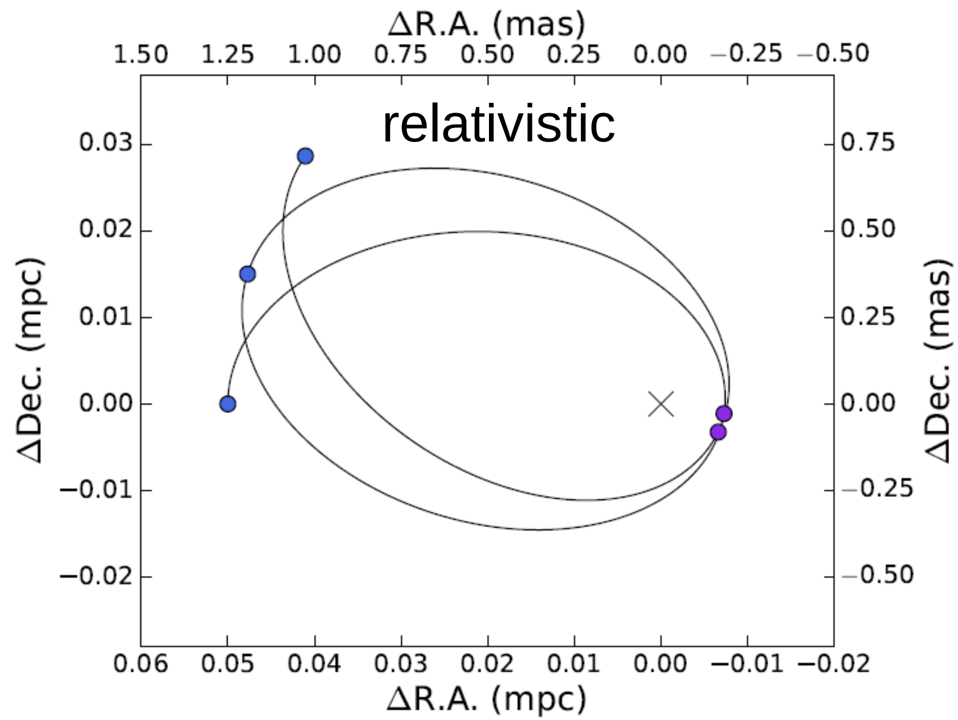
Elements can be parameterized by the relativistic parameter Y.
This parameter is attractive as it is proportional to the pericenter shift.

Parsa et al. (2017)

Method

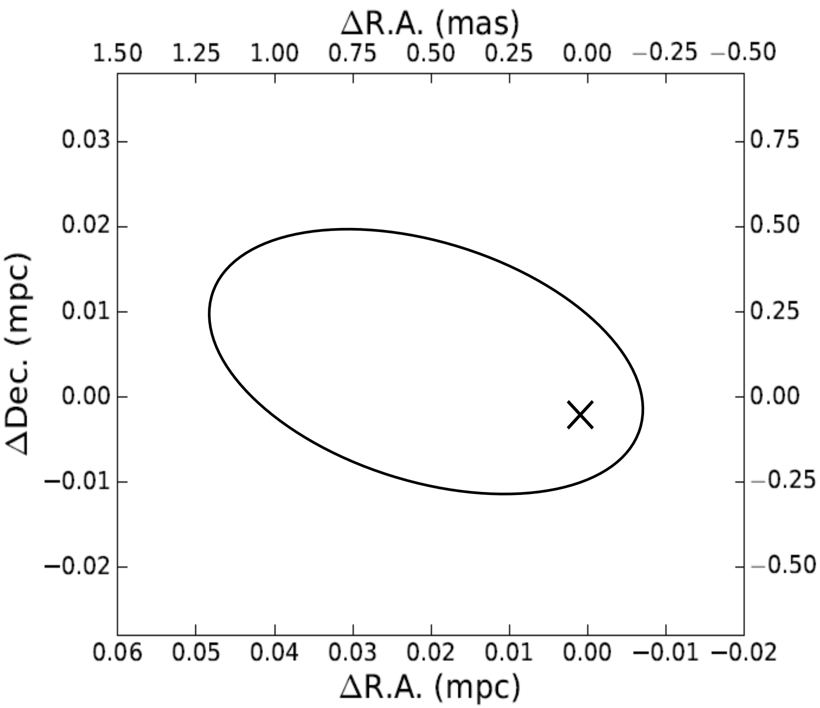


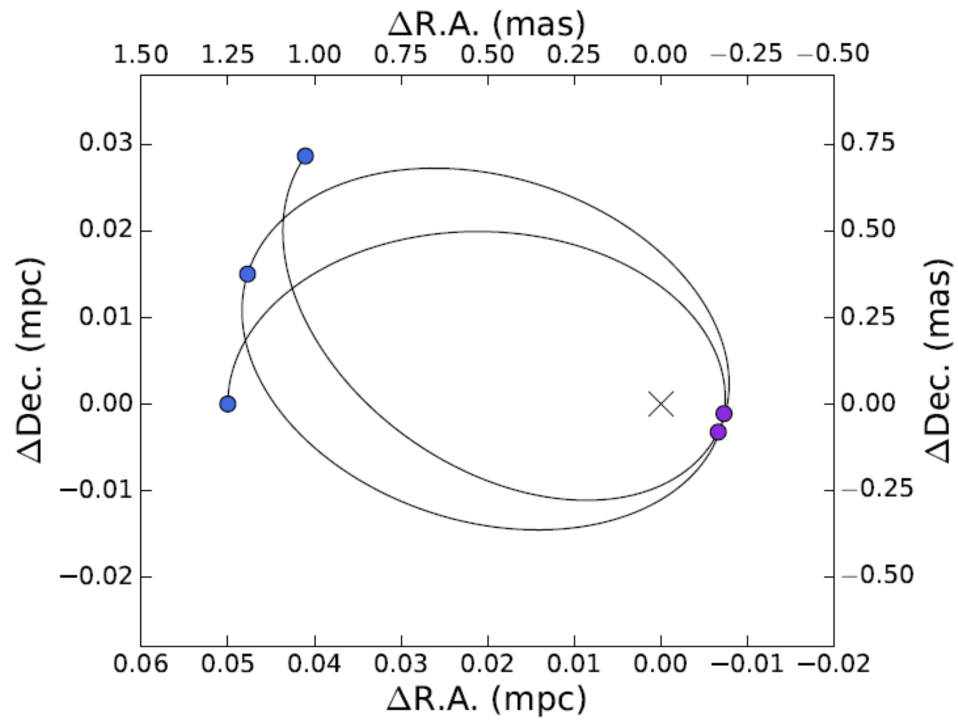
6 elements:
e,a,i,Ω,ω,t
negligible
computation time



Mass, 5-7 launching parameters,
Post-Newtonian formalism,
4th order Runge-Kutta method,
non-negligible computation time

Method



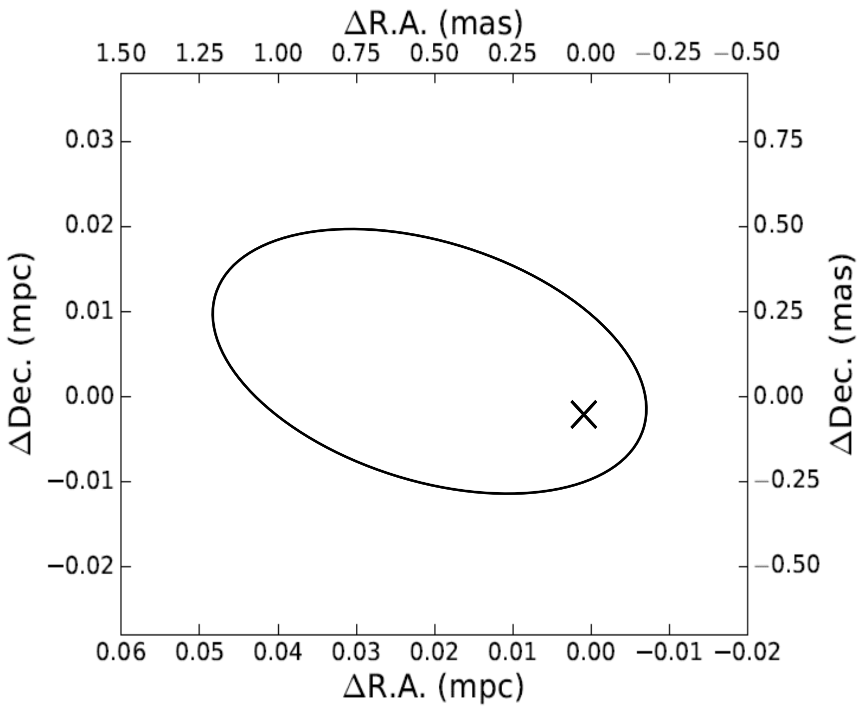


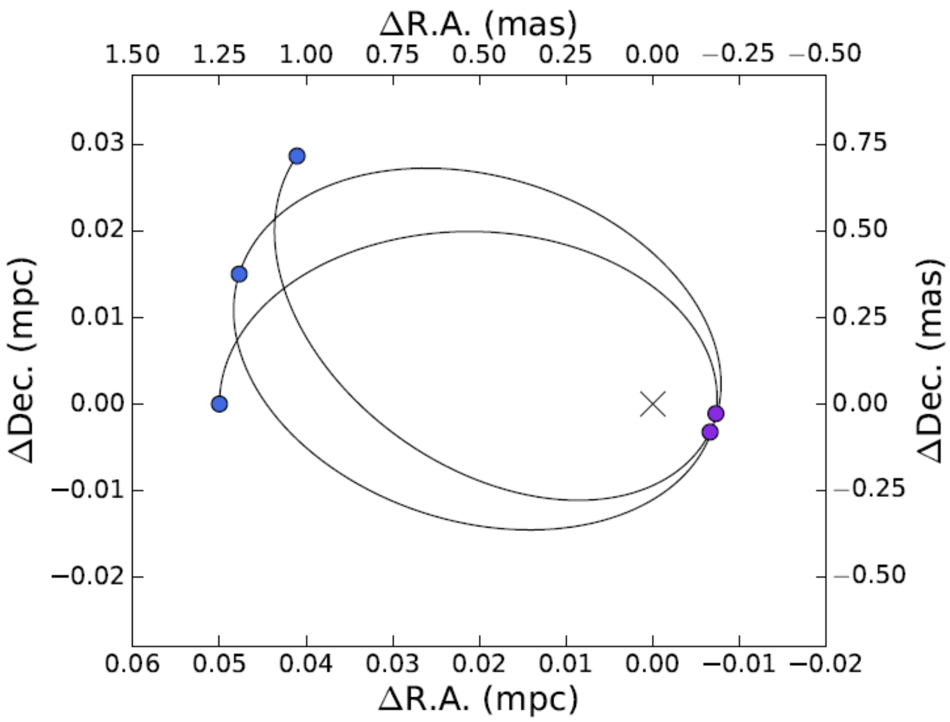
6 elements:
$e, a, i, \Omega, \omega, t$
negligible
computation time

Mass, 5-7 launching parameters,
Post-Newtonian formalism,
4th order Runge-Kutta method,
non-negligible computation time

Relativistic orbits can
not easily be parameterized

Method





6 elements:
e,a,i,Ω,ω,t
negligible
computation time

Mass, 5-7 launching parameters,
Post-Newtonian formalism,
4th order Runge-Kutta method,
non-negligible computation time

We need a simpler method to describe the relativistic character of an orbit.
Preferable by simple, non-relativistic orbit fitting
combined with a suitable parameterization.

Method

Squeezed states:

$$\alpha \times \beta \geq \varepsilon$$

For orbital fits:
l = lower part
u= upper part
of orbit
ul= overall fit

r = random
s = systematic
χ^2=fit parameter

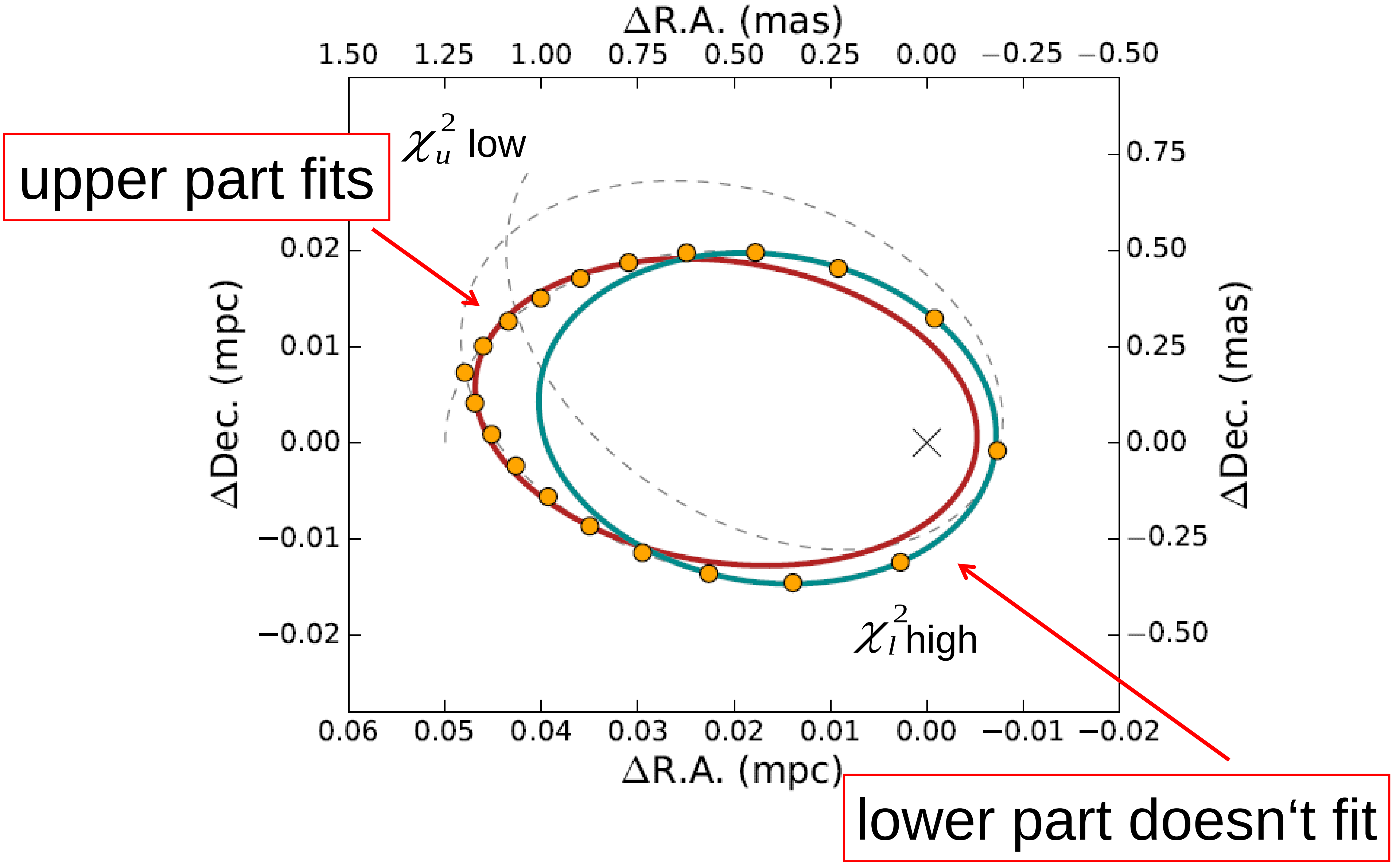
$$e^{-\chi_l^2} \times e^{-\chi_u^2} \geq e^{-\chi^2}$$

Fitting only one part of the orbit squeezes the bulk of the uncertainties into the other part.

$$\chi_{l,s}^2 + \chi_{u,s}^2 + \chi_{l,r}^2 + \chi_{u,r}^2 \geq \chi_{ul,s}^2 + \chi_{ul,r}^2$$

Random due to noise; systemetic due to **non ellipticity**

Method: the squeezing

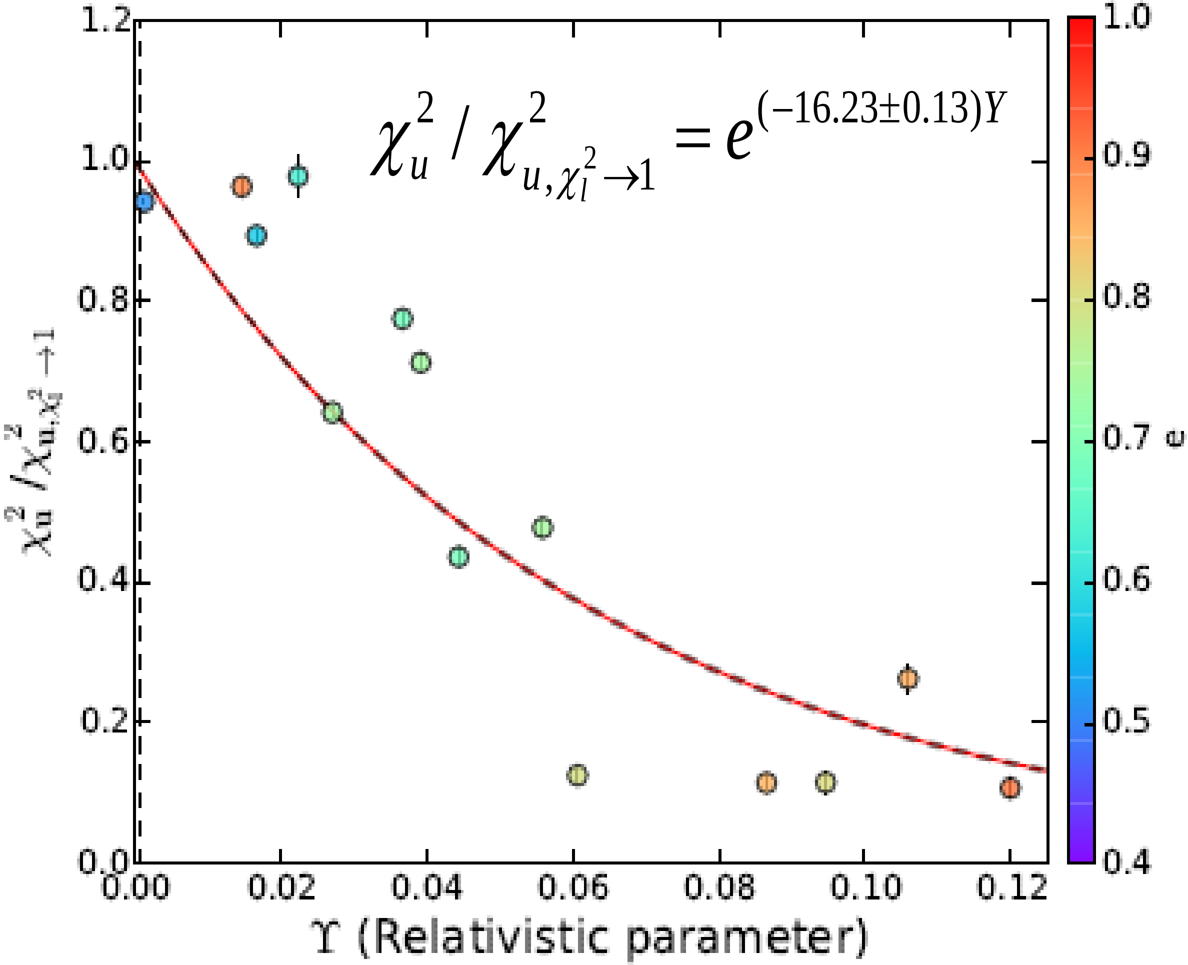


Method

mis-fit ratio:

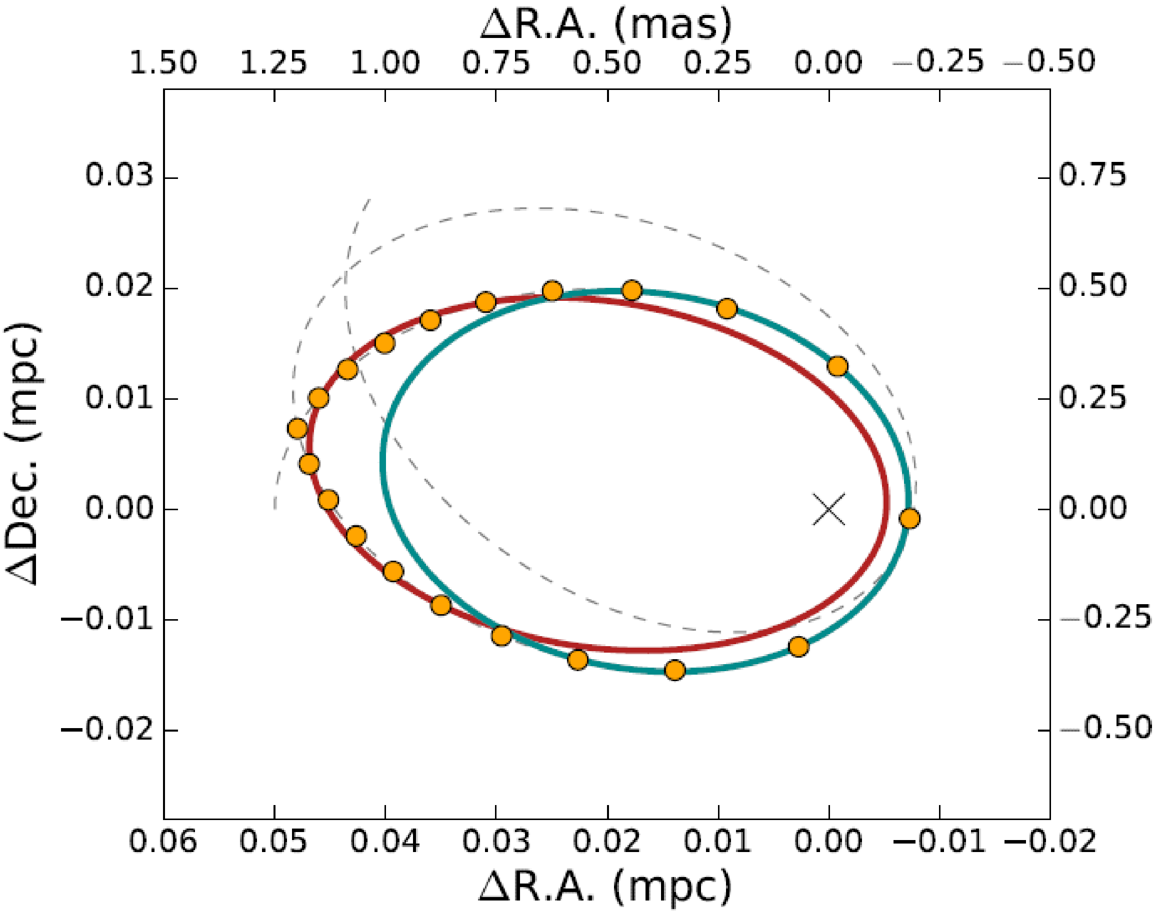
$$\chi_u^2 / \chi_{u,\chi_l^2 \to 1}^2$$

best fit on one side over fit on this side if fit on opposite side is optimized.



Method

Squeezing allows to derive measures for non ellipticity.
All of these quantities measure the deviation from ellipticity and will be correlated with the degree of relativity:



$$\chi_u^2 / \chi^2_{u,\chi_l^2 \to 1}$$

$$a_l / a_u$$

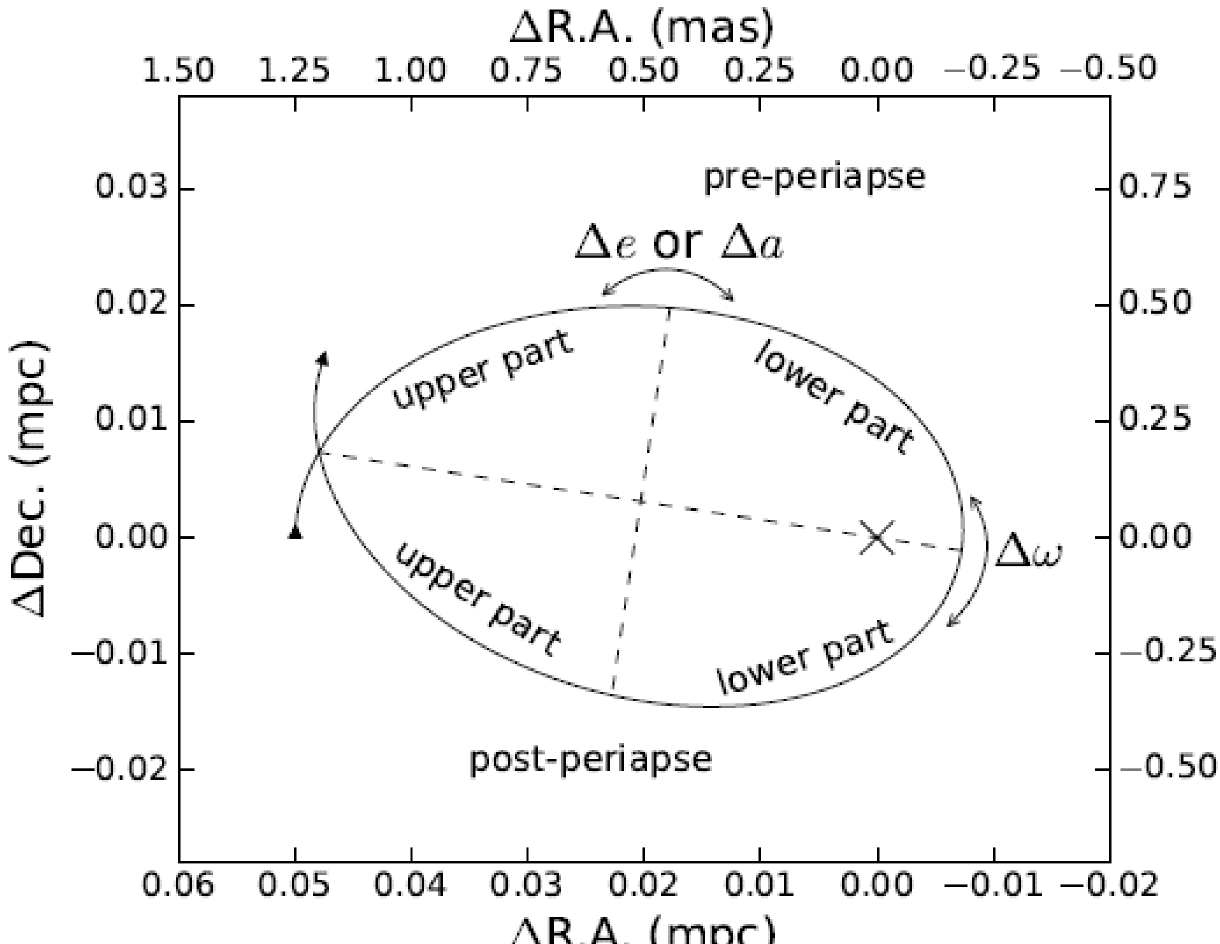
$$e_l / e_u$$

$$\Delta\omega$$

Method

Squeezing allows to easily derive measures for non ellipticity.

All of these quantities measure the deviation from ellipticity and will be correlated with the degree of relativity:



$$\chi_u^2 / \chi_{u,\chi_l^2 \to 1}^2$$

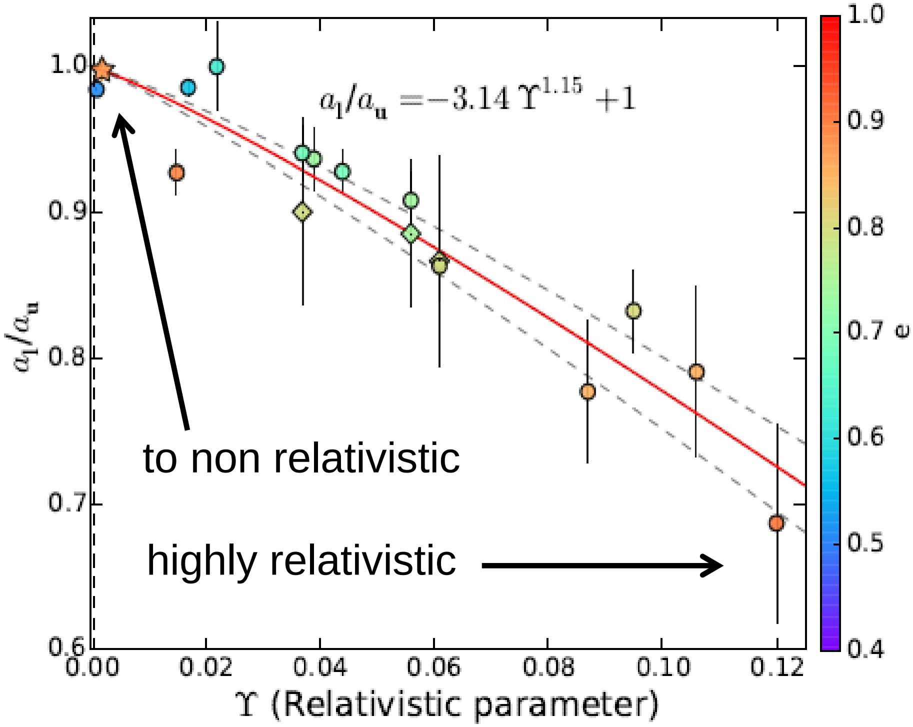
$$a_l / a_u$$

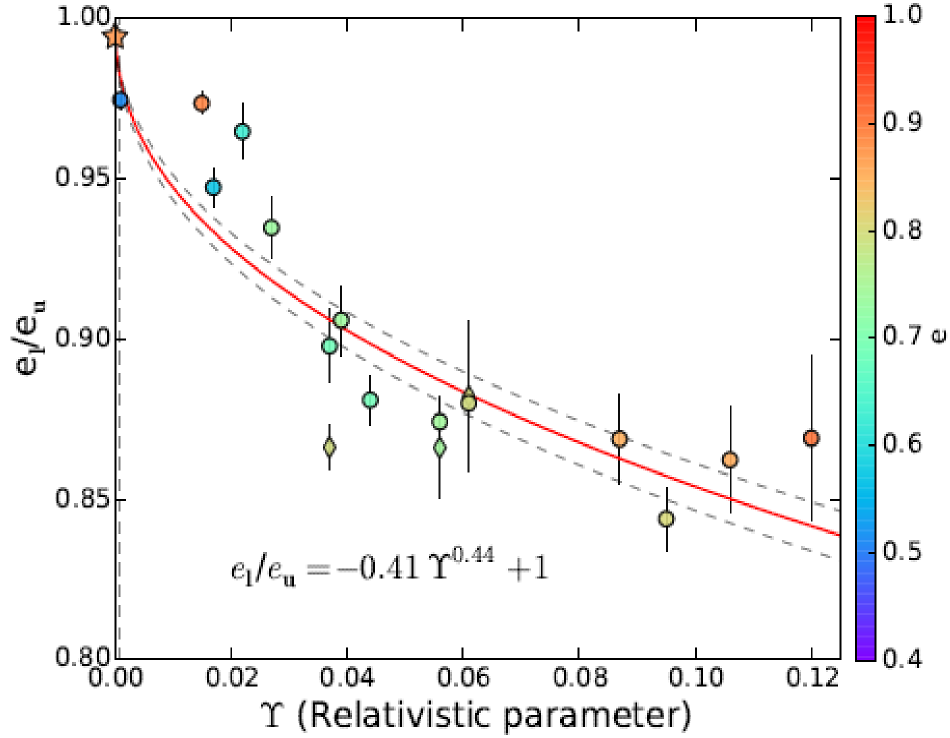
$$e_l / e_u$$

$$\Delta\omega$$

Results

Parameterizing a Measure of Relativity





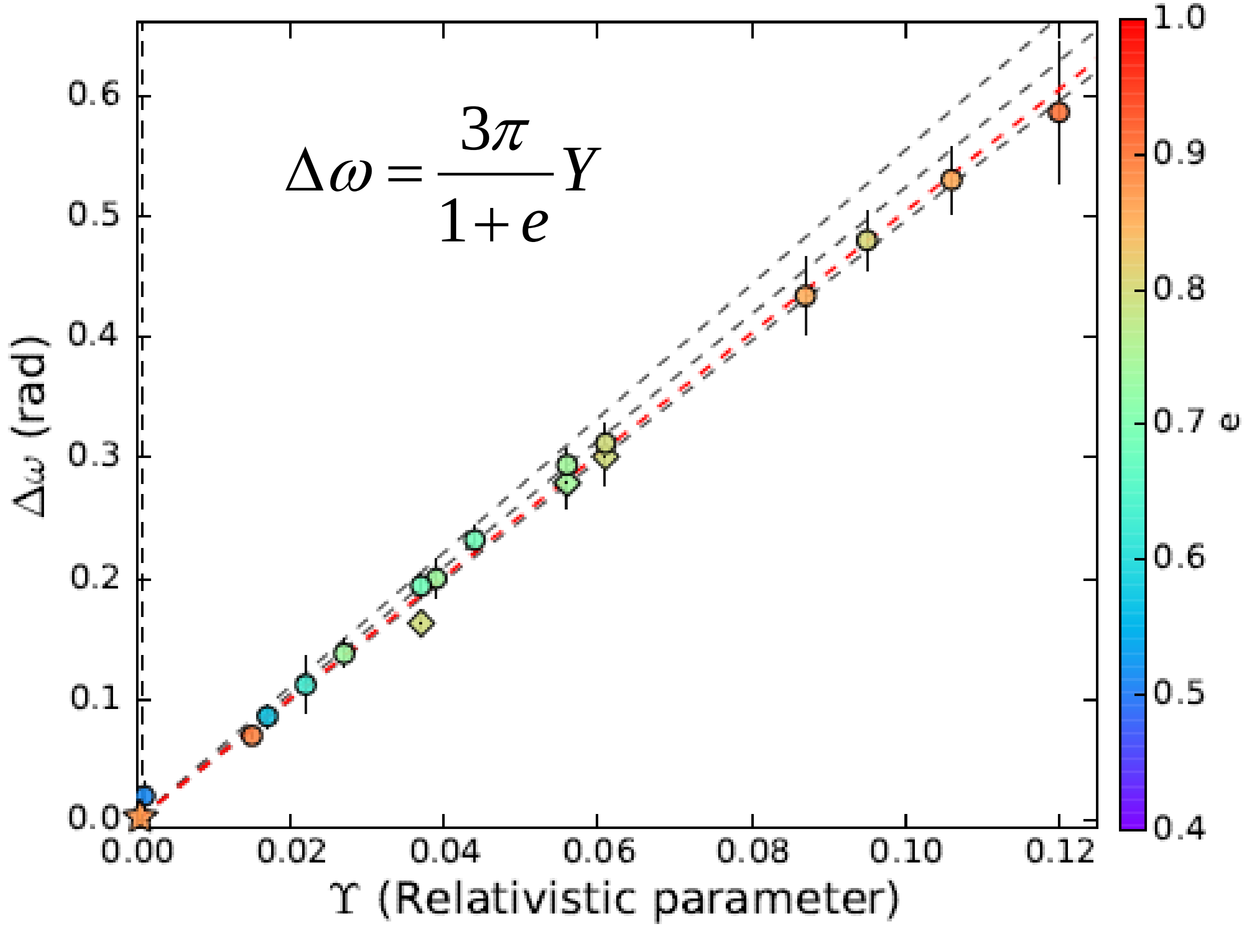
Relativistic Parameter Y:
Zucker et al. 2006

$$Y = \frac{r_s}{r_p}$$

r_s Schwarzschild radius; r_p periapse distance

Parsa et al. (2017)

Parameterizing a Measure of Relativity



Relativistic Parameter Y: Zucker et al. 2006

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Parsa et al. (2017)

Extracting information for S2

Question:

Is the current single dish AO data set of S2 accurate enough to show the effects of GR?

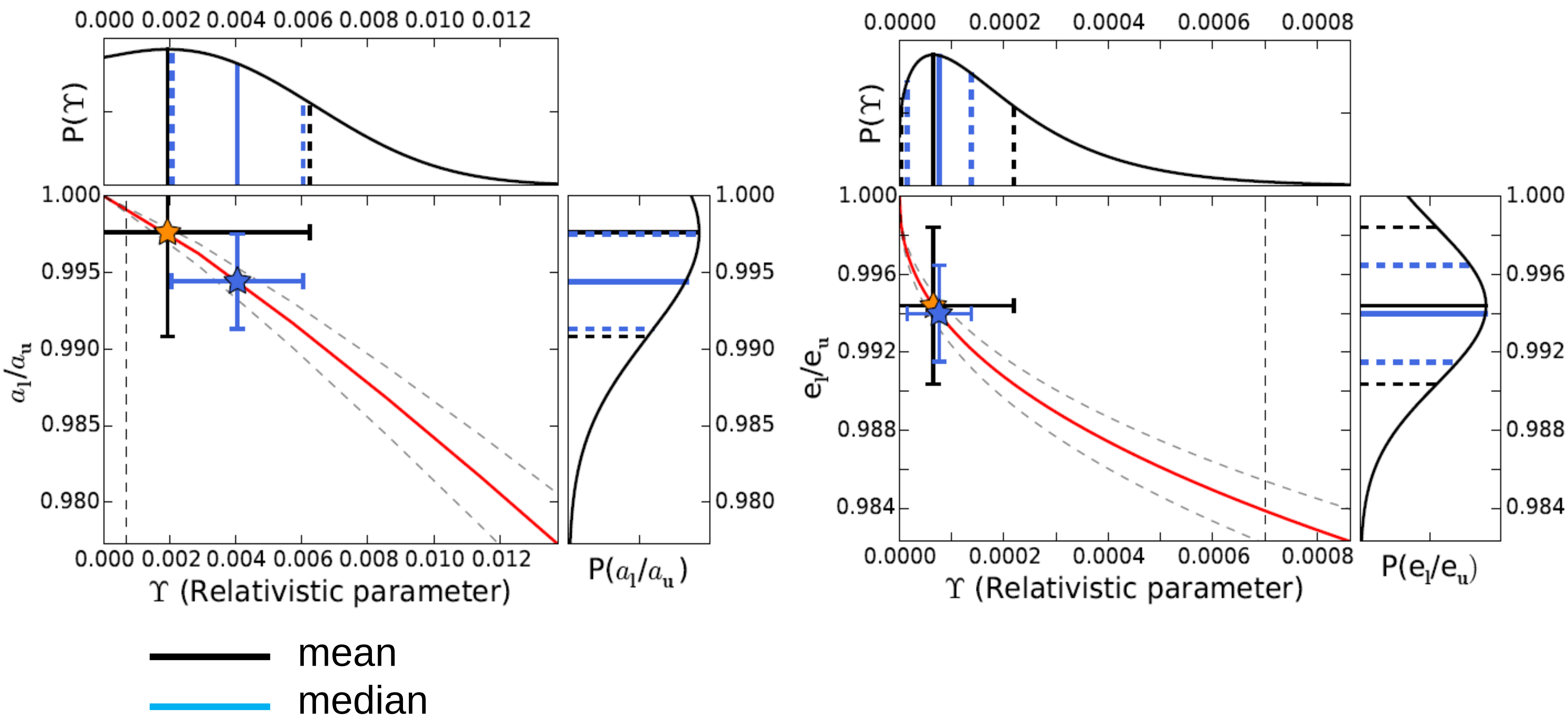
Procedure:

Measure off the a- and e-ratios

a_l / a_u and e_l / e_u

as well as $\Delta\omega$ compare with results from simulated stars.

Extracting information for S2



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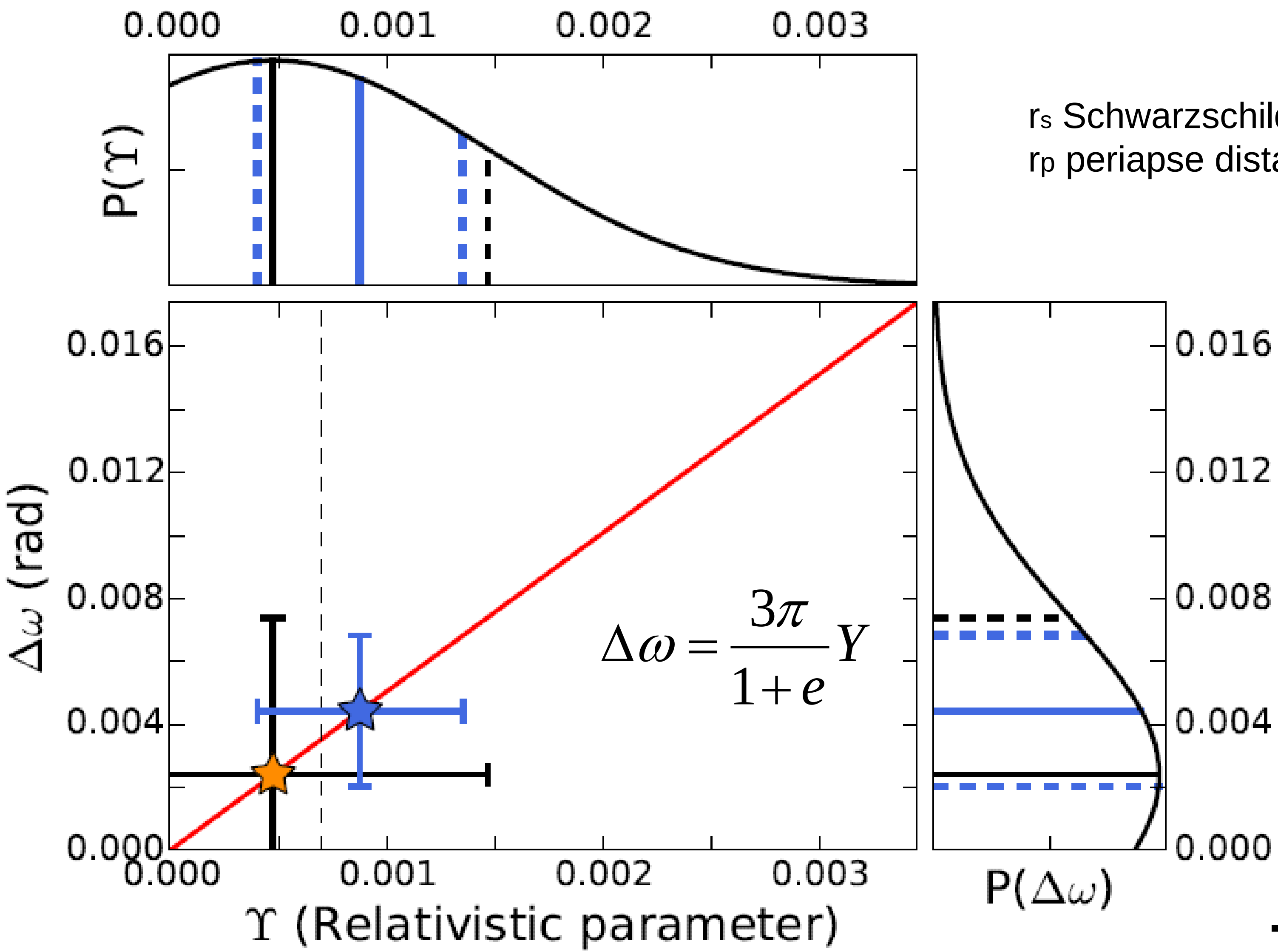


r_s Schwarzschild radius

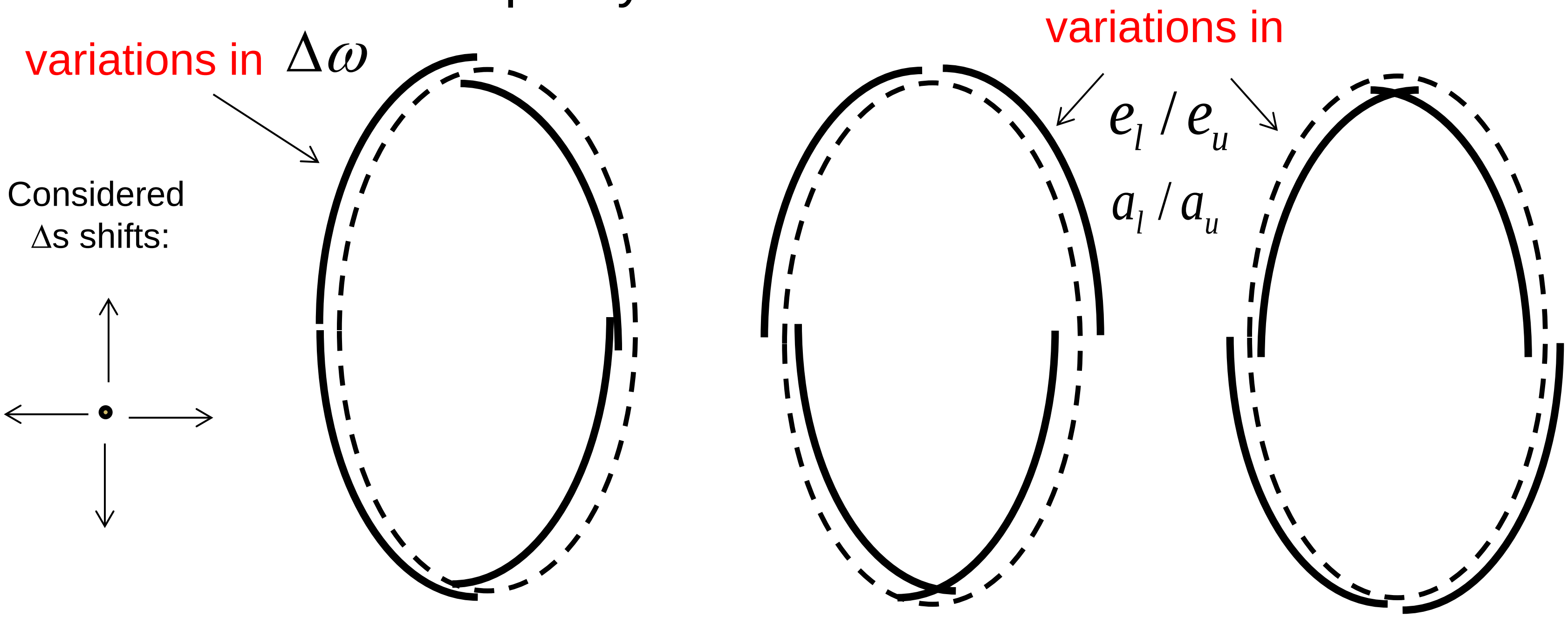
r_p periapse distance

Parsa et al. (2017)

How significant is the result really?

The uncetrainties for the e-, and a-ratios as well as the Δω value were obtained by transporting the uncertainties from the measurements, via the reference frames to the final statement.

As we used only images in which SgrA* could be detected as well, the positional uncertainties are the most important quantites in order to measure the non ellipticity.

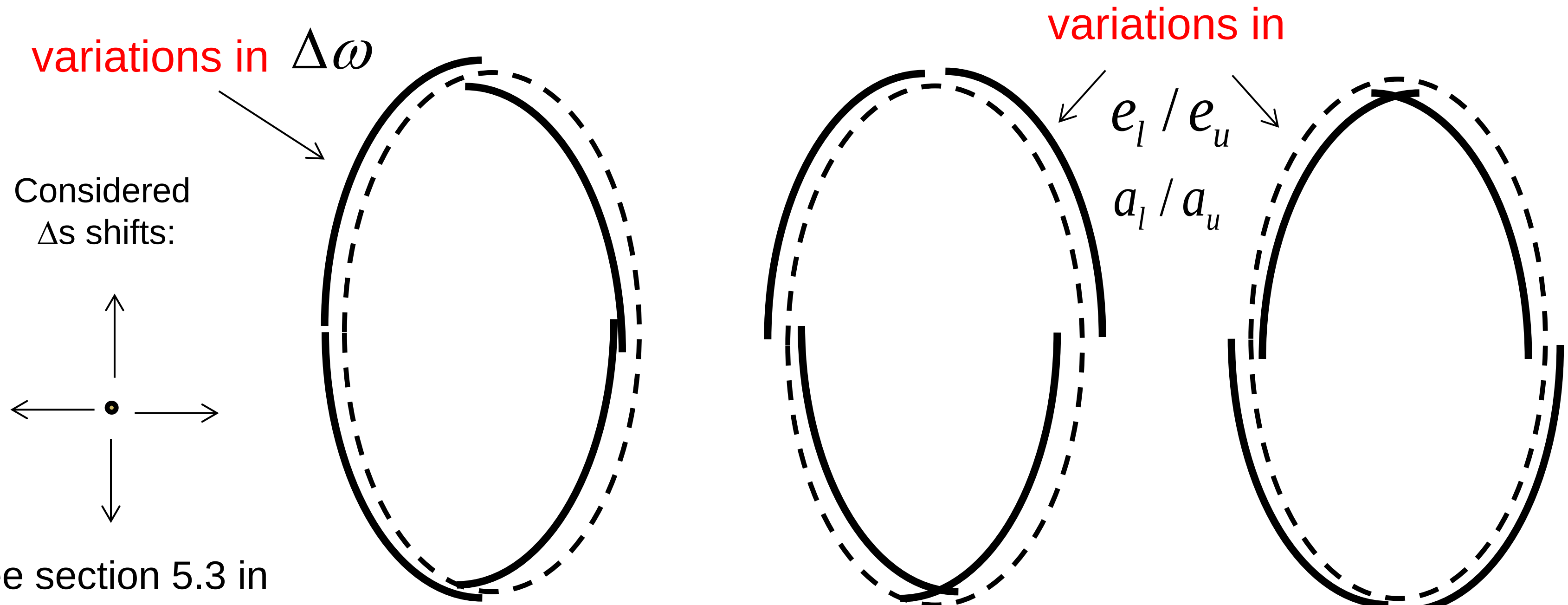


Estimating uncertainties relative to a noise dominated case

We use the combination of our uncertainty in R.A. direction (essential the $\Delta\omega$ mesurement of S2) and the literature data.
For an individual position we then find a mean uncertainty of 1.4 mas.

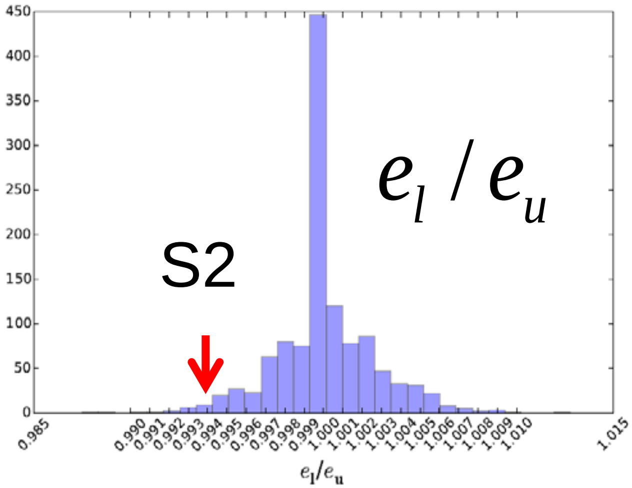
For about 7 data points per **<u>quarter of the orbit</u>** this corresponds to a positioning uncertainty of each quarter of about Δs = 0.5 mas.

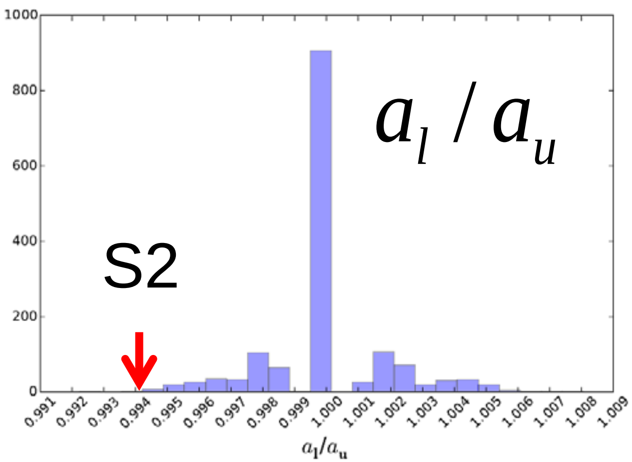
Rendomizing the position of the orbital segments with Δs=0,+0.5,-0.5 mas :

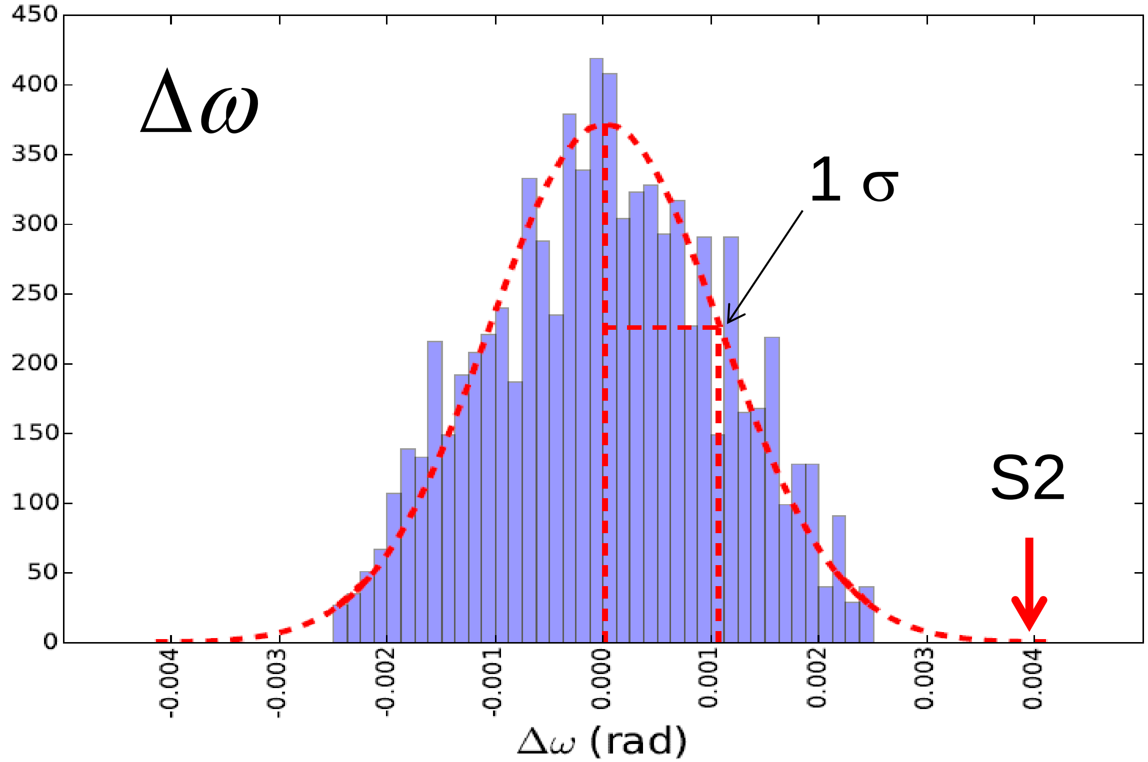


see section 5.3 in
Parsa et al. 2017

Estimating uncertainties relative to a noise dominated case

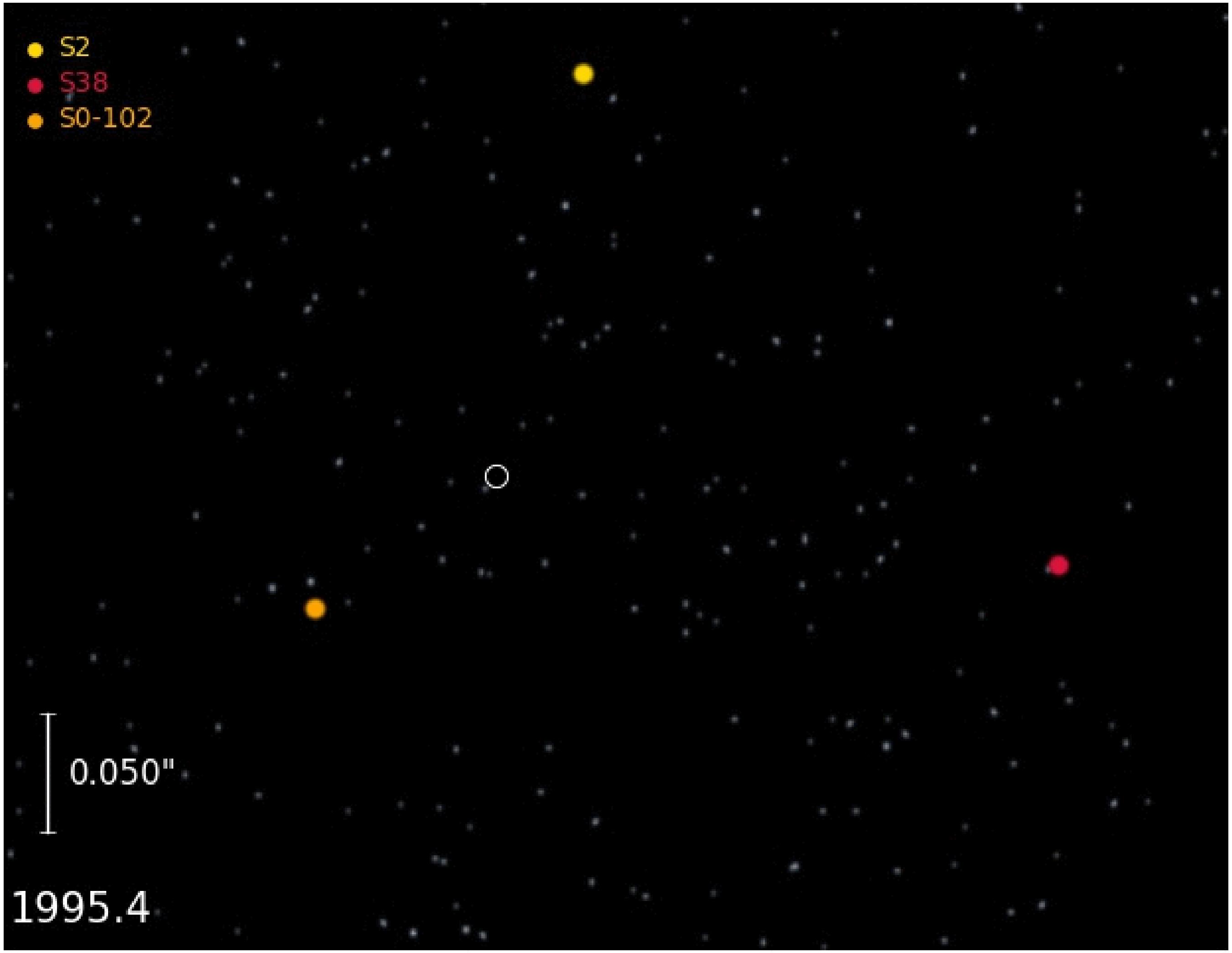






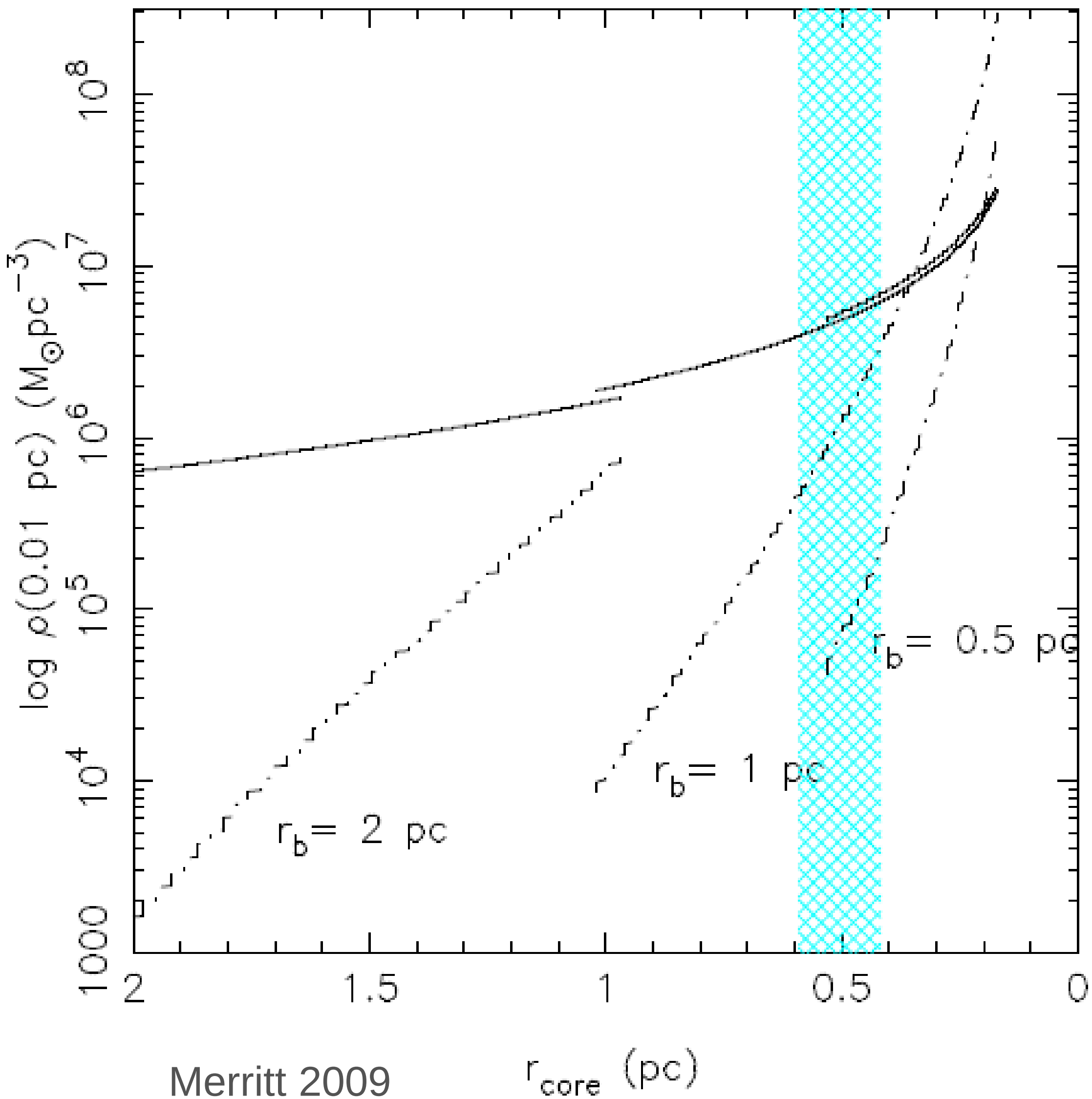
With respect to a noise dominated situation the S2 values for the e- and a-ratios and Δω represent **3-4σ excursions**.

Visualization of Results



ESO press annoncement 9 August 2017: ann17051:
Hint of Relativity Effects in Stars Orbiting Supermassive Black Hole at Centre of Galaxy

BH density in a dynamical core



Merritt 2009

The stellar BH density is expected to be largest at a radius of a few 0.1 pc.

Most authors claim a ~10 Msol population of black holes residing at the 'bottom' of the central potential well

Chandra observations by Muno, Baganoff + 2008, 2009

and simulations by Freitag et al. 2006 Merritt 2009

Histograms of the predicted peri-bothron change of S2 over one orbital period

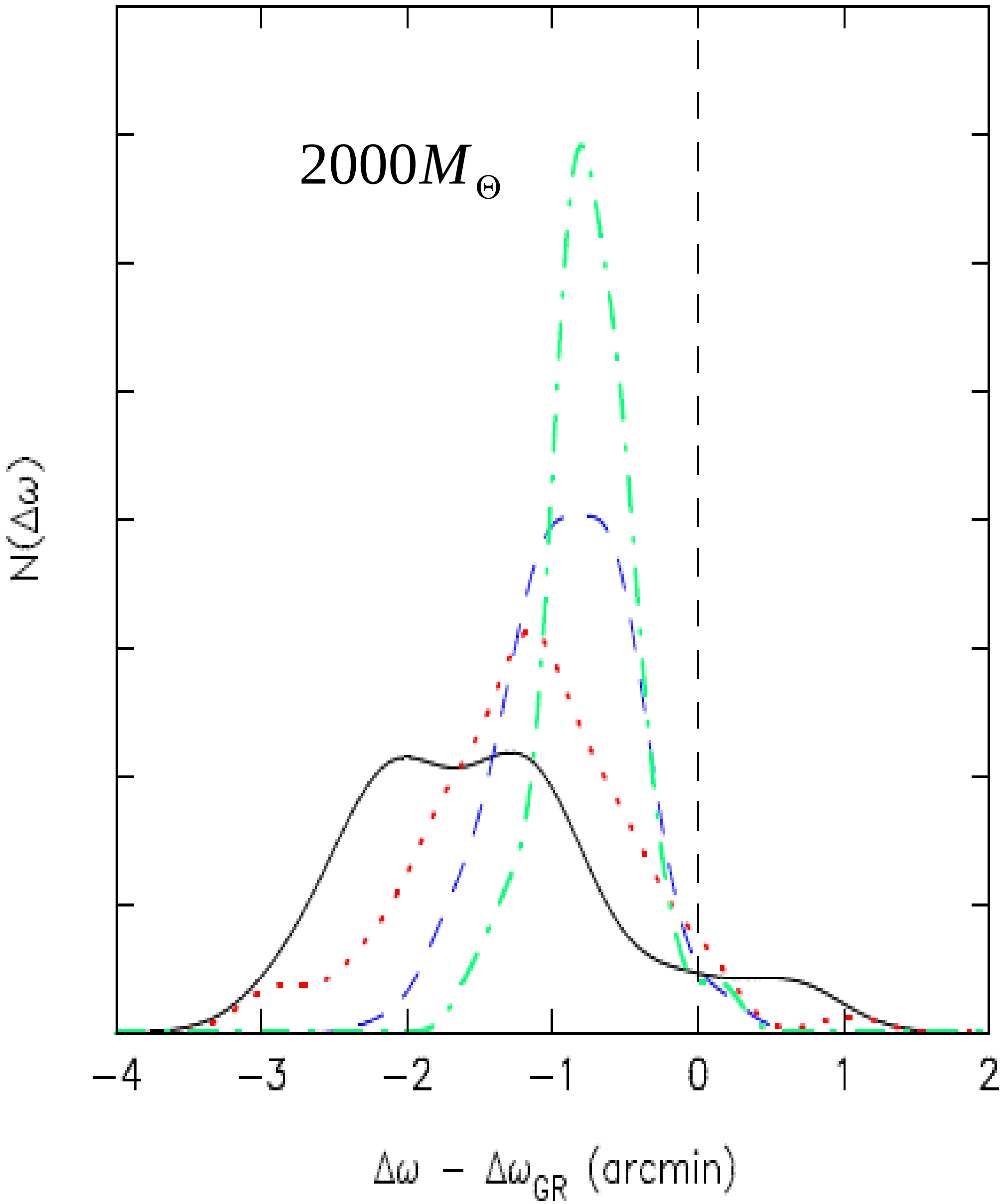
Perturbation/scattering can be as large as the entire expected Newtonian periastron shift.

Significant contributions to perisatron shift $\Delta\varpi$ from encounters due to granulartiy of 'scattering' Population and variation in enclosed mass due to scattering population:

$$\Delta\theta \propto \sqrt{N_{stellar-BHs}}$$

Higher accuracy needed to make first statement on scattering population. Massive IBMH can probably be excluded.

Sabha et al. 2012, A&A 545, 70



Results

The best estimates for the mass and the distance to Sgr A* are:

$$M_{BH} = (4.15 \pm 0.13 \pm 0.57) \times 10^{6} M_{sun}$$

$$R_0 = 8.19 \pm 0.11 \pm 0.34 \ kpc$$

conservative; probably more around 3‘

The change in the argument of periapse of S2 is:

S2 periapse:
2018.51 +- 0.22
which is in July

$$\Delta\omega_{obs} = 14' \pm 7'$$

$$\Delta\omega_{expected} = 11'$$

The changes in the orbital elements of S2 imply a relativistic parameter of:

$$Y_{obs} = 0.00088 \pm 0.00080$$

$$Y_{expected} = 0.00065$$

conservative; probably more around 0.0004

Summary

- We used three stars to derive the mass and distance of SgrA* in a Newtnian and post-Newtonian solution.
- We present a new and simple method that allows us through fits of simple ellipses to determin the degree of relativity.
- For S2 the values for the e- and a-ratios as well as $\Delta\omega$ value lie close to the values expected for S2 and the SgrA* mass.
- With respect to a noise dominated situation the S2 values for the e- and a-ratios and $\Delta\omega$ represent 3-4σ excursions.

Excepting this result, S2 is the first star with a resolvable orbit around a SMBH for which a test for relativity can be performed.
We all look forward to more high precision Keck and VLT as well as VLTI - GRAVITY results (see talk by Frank Eisenhauer)

End